Math 15900 Exam Jam

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Chapter 1: Fundamental Concepts of Algebra

1. Simplify and rationalize the denominator when appropriate.

\[
\frac{\sqrt[4]{5x^8y^3}}{27x^2}
\]

*Section 1.2 - Exponents and Radicals*

2. Factors the expressions.
   a. \(64x^3 - y^6\)
   b. \(y^2 - x^2 + 8y + 16\)

*Section 1.3 – Algebraic Expressions*

3. Simplify the following expression.

\[
\frac{(4x^2 + 9)^{\frac{1}{2}}(2) - (2x + 3)(\frac{1}{2})(4x^2 + 9)^{\frac{1}{2}}(8x)}{[(4x^2 + 9)^{\frac{1}{2}}]^2}
\]

*Section 1.4 – Fractional Expressions*
Chapter 2: Equations and Inequalities

4. Solve for $q$.

$$S = \frac{p}{q + p(1 - q)}$$

Section 2.1 – Equations

5. Solve the equation.

$$\frac{2}{2x + 1} - \frac{3}{2x - 1} = \frac{-2x + 7}{4x^2 - 1}$$

Section 2.1 – Equations

6. In a certain medical test designed to measure carbohydrate tolerance, an adult drinks 7 ounces of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

Section 2.2 – Applied Problems

7. Solve by completing the square.

$$4x^2 - 12x - 11 = 0$$

Section 2.3 – Quadratic Equations

8. Solve the equation.

$$\frac{3}{2}z^2 - 4z - 1 = 0$$

Section 2.3 – Quadratic Equations

9. A farmer plans to close a rectangular region, using part of his barn for one side and fencing for the other three sides. If the side parallel to the barn is to be twice the length of the adjacent side, and the area of the region is to be 128 ft$^2$. How many feet of fencing should be purchased?

Section 2.3 – Applied Problems
10. A baseball is thrown straight upward with an initial speed of 64 ft/sec. The number of feet $(s)$ above the ground after $t$ seconds is given by the equation 

\[ s = -16t^2 + 64t \]

a. When will the baseball be 48 feet above the ground?
b. When will the ball hit the ground?

Section 2.3 – Applied Problems

11. The recommended distance $d$ that a ladder is placed away from a vertical wall is 25% of its length $L$. Approximate the height $h$ that can be reached by relating $h$ as a percentage of $L$.

Section 2.3 – Applied Problems

12. Write in the form $a \pm bi$, where $a$ and $b$ are real numbers.

\[ \frac{-4 + 6i}{2 + 7i} \]

Section 2.4 – Complex Numbers

13. Find the values of $x$ and $y$, where $x$ and $y$ are real numbers.

\[(2x - y) - 16i = 10 + 4yi\]

Section 2.4 – Complex Numbers

14. Find the solutions to the equation.

\[ 4x^4 + 25x^2 + 36 = 0 \]

Section 2.4 – Complex Numbers

15. Solve the equation.

\[ f(x) = 2x^{-\frac{2}{3}} - 7x^{-\frac{1}{3}} - 15 = 0 \]

Section 2.5 – Other Types of Equations
16. Solve for $x$.

$$x = 4 + \sqrt{4x - 19}$$

*Section 2.5 – Other Types of Equations*

17. Solve the equation for $x$.

$$3|x + 1| - 2 = -11$$

*Section 2.5 – Other Types of Equations*

18. Solve and express the solutions in terms of intervals if possible.

$$-\frac{1}{3}|6 - 5x| + 2 \geq -1$$

*Section 2.6 – Inequalities*

19. Solve and express the solutions in terms of intervals if possible.

$$\frac{3}{|5 - 2x|} < 2$$

*Section 2.6 – Inequalities*

20. Solve and express the solution in terms of intervals if possible.

$$\frac{x - 2}{x^2 - 3x - 10} \geq 0$$

*Section 2.7 – More on Inequalities*

21. Solve and express the solution in terms of intervals if possible.

$$\frac{x + 1}{2x - 3} > 2$$

*Section 2.7 – More on Inequalities*
Chapter 3: Functions and Graphs

22. Find an equation of the circle that has the endpoints of its diameter at \( A(4, -3) \) and \( B(-2, 7) \).

Section 3.2 – Graphs of Equations

23. Find the center and radius of the circle with the given equation.

\[
2x^2 + 2y^2 - 12x + 4y - 15 = 0
\]

Section 3.2 – Graphs of Equations

24. Find the general form of the equation for the perpendicular bisector of the segment \( AB \).

\[
A(3, -1) \quad B(-2, 6)
\]

Section 3.3 – Lines

25. Simplify the difference quotient when \( f(x) = x^2 + 5 \).

\[
\frac{f(x + h) - f(x)}{h} \quad \text{for } h \neq 0
\]

Section 3.4 – Definition of Function

26. Find the domain and sketch the graph.

\[
f(x) = \begin{cases} 
  x + 9 & \text{if } x < -3 \\
  -2x & \text{if } |x| \leq 3 \\
  -6 & \text{if } x > 3
\end{cases}
\]

Section 3.5 – Graphs of Functions

27. Determine whether \( f \) is even, odd, or neither.

\[
f(x) = 8x^3 - 3x^2
\]

Section 3.5 – Graphs of Functions
28. Express $f(x)$ in the form $f(x) = a(x - h)^2 + k$ and graph.

$$f(x) = -3x^2 - 6x - 5$$

*Section 3.6 – Quadratic Functions*

29. For $f(x) = \sqrt{3 - x}$ and $g(x) = \sqrt{x^2 - 16}$, find

a. $(f \circ g)(x)$ and its domain

b. $(g \circ f)(x)$ and its domain

*Section 3.7 – Operations on Functions*
Chapter 4: Polynomial and Rational Functions

30. Find all $x$ such that $f(x) > 0$ and all $x$ such that $f(x) < 0$. Then sketch the graph of $f$.

Section 4.1 – Polynomial Functions of Degree Greater Than 2

31. Find the quotient and remainder if $f(x)$ is divided by $p(x)$.

$$f(x) = 3x^3 + 2x - 4 \quad p(x) = 2x^2 + 1$$

Section 4.2 – Properties of Division

32. Use synthetic division to find the quotient and remainder if $f(x)$ is divided by $p(x)$.

$$f(x) = 2x^3 - 3x^2 + 4x - 5 \quad p(x) = x - 2$$

Section 4.2 – Properties of Division
Chapter 5: Inverse, Exponential, and Logarithmic Functions

33. Find the inverse function of \( f(x) = \frac{3x+2}{2x-5} \).

Section 5.1 – Inverse Functions

34. Determine the domain and range of \( f^{-1} \) for the given function.
\[
f(x) = -\frac{4x + 5}{3x - 8}
\]

Section 5.1 – Inverse Functions

35. If $1000 is invested at a rate of 7% per year compounded monthly, find the amount after:
   a. 1 month
   b. 6 months
   c. 1 year
   d. 20 years

Section 5.2 – Exponential Functions

36. If \( P \) dollars is deposited in a savings account that pays interest at a rate of \( r \)% per year compounded continuously, find the balance after \( t \) years.
\[
P = 1000 \quad r = 8 \frac{1}{4} \quad t = 5
\]

Section 5.3 – The Natural Exponential Functions

37. Find the zeros of \( f(x) \).
\[
f(x) = x^3 \left( 4e^{4x} \right) + 3x^2 e^{4x}
\]

Section 5.3 – The Natural Exponential Functions

38. The population \( N(t) \) (in millions) of the United States \( t \) years after 1980 may be approximated by the formula:
\[
N(t) = 231e^{0.0103t}
\]

When will the population be twice what it was in 1980?

Section 5.4 – Logarithmic Functions
39. Solve for $t$ using logarithms with base $a$.

$$A = B a^{Ct} + D$$

Section 5.4 – Logarithmic Functions

40. Solve the equation.

$$\ln(-4 - x) + \ln 3 = \ln (2 - x)$$

Section 5.5 – Properties of Logarithms

41. Solve the equation.

$$\log_3(x + 3) + \log_3(x + 5) = 1$$

Section 5.5 – Properties of Logarithms

42. Use natural logarithms to solve for $x$ in terms of $y$.

$$y = \frac{e^x - e^{-x}}{2}$$

Section 5.6 – Exponential and Logarithmic Equations

43. Solve the equations.

a. $3^{x+4} = 2^{1-3x}$

b. $2^{2x-3} = 5^{x-2}$

Section 5.6 – Exponential and Logarithmic Equations

44. Find the exact solution, using common logarithms, and two-decimal-place approximation, when appropriate.

$$\log(x - 4) - \log(3x - 10) = \log \left( \frac{1}{x} \right)$$

Section 5.6 – Exponential and Logarithmic Equations

45. Find the exact solution, using common logarithms, and two-decimal-place approximation, when appropriate.

$$4^x - 3(4^{-x}) = 8$$

Section 5.6 – Exponential and Logarithmic Equations
Chapter 6: The Trigonometric Functions

46. Given \( s = 7 \) cm and \( r = 4 \) cm, answer the following
   a. Find the radian and degree measures of the central angle \( \theta \) subtended by the given arc
      of length \( s \) on a circle of radius \( r \).
   b. Find the area of the sector determined by \( \theta \).

   \( \text{Section 6.1 – Angles} \)

47. Given a radius 5 in. and 40 rpm, answer the following:
   a. Find the angular speed (in radians per minute).
   b. Find the linear speed of a point on the circumference (in feet per minute).

   \( \text{Section 6.1 – Angles} \)

48. Use the fundamental identities to write \( \cot \theta \) in terms of \( \sin \theta \) for any acute angle \( \theta \).

   \( \text{Section 6.2 – Trigonometric Functions of Angles} \)

49. Verify the identity.
   \[ \sec(\theta) - \cos(\theta) = \tan(\theta) \sin(\theta) \]

   \( \text{Section 6.2 – Trigonometric Functions of Angles} \)

50. Find the exact value.
   a. \( \csc \left( \frac{3\pi}{4} \right) \)
   b. \( \csc \left( -\frac{2\pi}{3} \right) \)

   \( \text{Section 6.4 – Values of Trigonometric Functions} \)

51. Approximate, to the nearest 0.01 radian, all angles \( \theta \) in the interval \([0,2\pi]\) that satisfy the
    equation.
   a. \( \sin(\theta) = 0.4195 \)
   b. \( \tan(\theta) = -3.2504 \)
   c. \( \sec(\theta) = 1.7452 \)

   \( \text{Section 6.4 – Values of Trigonometric Functions} \)
52. Find the amplitude, period, and phase shift, then sketch the graph.

\[ y = -2 \sin(3x - \pi) \]

Section 6.5 – Trigonometric Graphs

53. An airplane takes off at a 10° angle and travels at a rate of 250 ft/sec. Approximately how long does it take the plane to reach an altitude of 15,000 ft?

Section 6.5 – Trigonometric Graphs
Chapter 7: Analytic Trigonometry

54. Verify the identity.

\[
\frac{1}{1 - \cos(\gamma)} + \frac{1}{1 + \cos(\gamma)} = 2\csc^2(\gamma)
\]

Section 7.1 – Verifying Trigonometric Identities

55. Verify the identity.

\[
\tan^4(k) - \sec^4(k) = 1 - 2\sec^2(k)
\]

Section 7.1 – Verifying Trigonometric Identities

56. Find all solutions to the equation.

\[
\sin \left(2x - \frac{\pi}{3}\right) = \frac{1}{2}
\]

Section 7.2 – Trigonometric Equations

57. Find the solutions that are in the interval \([0, 2\pi)\).

\[
2\tan(t) - \sec^2(t) = 0
\]

Section 7.2 – Trigonometric Equations

58. Approximate, to the nearest 10', the solutions in the interval \([0^\circ, 360^\circ)\).

\[
\sin^2(t) - 4\sin(t) + 1 = 0
\]

Section 7.2 – Trigonometric Equations
59. If \( \sin(\alpha) = -\frac{4}{5} \) and \( \sec(\beta) = \frac{5}{3} \) for a third-quadrant angle \( \alpha \) and a first-quadrant angle \( \beta \), find the following.
   a. \( \sin(\alpha + \beta) \)
   b. \( \tan(\alpha + \beta) \)
   c. The quadrant containing \( \alpha + \beta \)

Section 7.3 – The Addition and Subtraction Formulas

60. Find the exact values of \( \sin 2\theta, \cos 2\theta, \) and \( \tan 2\theta \) given the information below.

\[
\sec \theta = -3 \\
90^\circ < \theta < 180^\circ
\]

Section 7.4 – Multiple-Angle Formulas

61. Find the solutions that are in the interval \([0, 2\pi)\).

\[
\sin 2t + \sin t = 0
\]

Section 7.4 – Multiple-Angle Formulas

62. Find the exact value.
   a. \( \cot(\sin^{-1} \frac{2}{3}) \)
   b. \( \sec(\tan^{-1}(-\frac{3}{5})) \)
   c. \( \csc(\cos^{-1}(-\frac{1}{4})) \)

Section 7.6 – The Inverse Trigonometric Functions

63. Find the exact value whenever it is defined.
   a. \( \sin(\arcsin(\frac{1}{2}) + \arccos(0)) \)
   b. \( \cos [\arctan(-\frac{3}{4}) - \arcsin(\frac{4}{5})] \)
   c. \( \tan(\arctan(\frac{4}{3}) + \arccos(\frac{8}{17})) \)

Section 7.6 – The Inverse Trigonometric Functions

64. Find the exact value whenever it is defined.
   a. \( \sin[2 \arccos(-\frac{3}{5})] \)
   b. \( \cos [2 \sin^{-1}(\frac{15}{17})] \)
   c. \( \tan [2 \tan^{-1}(\frac{3}{4})] \)

Section 7.6 – The Inverse Trigonometric Functions
65. Write the expression as an algebraic expression in terms of $x$ for $x > 0$.

$$
\sin(2 \sin^{-1} x)
$$

*Section 7.6 – The Inverse Trigonometric Functions*

66. Use inverse trigonometric functions to find the solutions of the equation that are in $[0, 2\pi)$. Approximate to four decimal places.

$$
\cos^2 x + 2 \cos x - 1 = 0
$$

*Section 7.6 – The Inverse Trigonometric Functions*
Chapter 8: Applications of Trigonometry

67. Solve ΔABC.
\[ \gamma = 81^\circ \quad c = 11 \quad b = 12 \]

Section 8.1 – The Law of Sines

68. A forest ranger at an observation point A sights a fire in the direction N27°10′ E. Another ranger at an observation point B, 6 miles east of point A, sights the same fire in the direction N52°40′ W. Approximate the distance between point A and the fire.

Section 8.1 – The Law of Sines

69. Solve ΔABC.
\[ a = 25.0 \quad b = 80.0 \quad c = 60.0 \]

Section 8.2 – The Law of Cosines

70. A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.

Section 8.2 – The Law of Cosines

71. Approximate the area of ΔABC.
\[ a = 80.1^\circ \quad a = 8.0 \quad b = 3.4 \]

Section 8.2 – The Law of Cosines

72. Approximate the area of ΔABC.
\[ a = 25.0 \quad b = 80.0 \quad c = 60.0 \]

Section 8.2 – The Law of Cosines

73. Approximate the areas of the parallelogram that has sides of length a and b (n feet) of one angle at the vertex has measure θ
\[ a = 12.0 \quad b = 16.0 \quad \theta = 40^\circ \]

Section 8.2 – The Law of Cosines
Chapter 9: Systems of Equations and Inequalities

74. Use the method of substitution to solve the system.

\[
\begin{align*}
    x^2 + y^2 &= 16 \\
    2y - x &= 4
\end{align*}
\]

Section 9.1 – Systems of Equations

75. Use the method of substitution to solve the system.

\[
\begin{align*}
    y^2 + 4x^2 &= 4 \\
    9y^2 + 16x^2 &= 140
\end{align*}
\]

Section 9.1 – Systems of Equations

76. Use the method of substitution to solve the system.

\[
\begin{align*}
    x &= y^2 - 4y + 5 \\
    x - y &= 1
\end{align*}
\]

Section 9.1 – Systems of Equations

77. The price of admission to a high school play was $3.00 for students and $4.50 for nonstudents. If 450 tickets were sold for a total of $1555.50, how many of each kind were purchased?

Section 9.2 – Systems of Linear Equations in Two Variables

78. A small furniture company manufactures sofas and recliners. Each sofa requires 8 hours of labor and $180 in materials, while a recliner can be built for $105 in 6 hours. The company has 340 hours of labor available each week and can afford to buy $6750 worth of materials. How many recliners and sofas can be produced if all labor hours and all materials must be used?

Section 9.2 – Systems of Linear Equations in Two Variables
Chapter 11: Topics from Analytic Geometry

79. Find the vertex, focus, and directrix of the parabola. Sketch its graph, showing the focus and the directrix.

\[ y = x^2 - 4x + 2 \]

\( \text{Section 11.1 – Parabolas} \)

80. Find an equation of the parabola that satisfies the given conditions.

Focus: \( F(6,4) \)
Directrix: \( y = -2 \)

\( \text{Section 11.1 – Parabolas} \)

81. Find the equation of the parabola that satisfies the given conditions.

Vertex \( V(-3,5) \), axis parallel to the x-axis, and passing through the point \( (5,9) \).

\( \text{Section 11.1 – Parabolas} \)

82. Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

\[ \frac{(x - 3)^2}{16} + \frac{(y + 4)^2}{9} = 1 \]

\( \text{Section 11.2 – Ellipses} \)

83. Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

\[ 4x^2 + 9y^2 - 32x - 36y + 64 = 0 \]

\( \text{Section 11.2 – Ellipses} \)

84. Find an equation of the ellipse that has its center at the origin and satisfies the given conditions:

Vertices: \( V(\pm 8,0) \)
Foci: \( F(\pm 5,0) \)

\( \text{Section 11.2 – Ellipses} \)
85. Find the vertices, the foci, and the equation of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

\[ 4y^2 - x^2 + 40y - 4x + 60 = 0 \]

Section 11.3 – Hyperbolas

86. Find the equation of the hyperbola that has its center at the origin and satisfies the given conditions:

- Vertices: \( V(\pm 4, 0) \)
- Passing through \((8, 2)\)

Section 11.3 – Hyperbolas