Math 15400 Exam Jam

Chapter 6: The Trigonometric Functions ........................................................................................................1

Chapter 7: Analytic Trigonometry ..................................................................................................................3

Chapter 8: Applications of Trigonometry .....................................................................................................6

Chapter 9: Systems of Equations and Inequalities .......................................................................................8

Chapter 11: Topics from Analytic Geometry .................................................................................................9

Chapter 6: The Trigonometric Functions

1. Given \( s = 7 \text{ cm} \) and \( r = 4 \text{ cm} \), answer the following
   a. Find the radian and degree measures of the central angle \( \theta \) subtended by the given arc of length \( s \) on a circle of radius \( r \).
   b. Find the area of the sector determined by \( \theta \).

   \( \text{Section 6.1 – Angles} \)

2. Given a radius 5 in. and 40 rpm, answer the following:
   a. Find the angular speed (in radians per minute).
   b. Find the linear speed of a point on the circumference (in feet per minute).

   \( \text{Section 6.1 – Angles} \)

3. Use the fundamental identities to write \( \cot \theta \) in terms of \( \sin \theta \) for any acute angle \( \theta \).

   \( \text{Section 6.2 – Trigonometric Functions of Angles} \)

4. Verify the identity.
   \[
   \sec(\theta) - \cos(\theta) = \tan(\theta) \sin(\theta)
   \]

   \( \text{Section 6.2 – Trigonometric Functions of Angles} \)

5. Find the exact value.
   a. \( \csc\left(\frac{3\pi}{4}\right) \)
   b. \( \csc\left(-\frac{2\pi}{3}\right) \)

   \( \text{Section 6.4 – Values of Trigonometric Functions} \)

6. Approximate, to the nearest 0.01 radian, all angles \( \theta \) in the interval \([0,2\pi]\) that satisfy the equation.
   a. \( \sin(\theta) = 0.4195 \)
   b. \( \tan(\theta) = -3.2504 \)
   c. \( \sec(\theta) = 1.7452 \)

   \( \text{Section 6.4 – Values of Trigonometric Functions} \)
7. Find the amplitude, period, and phase shift. Then sketch the graph.

\[ y = -2 \sin(3x - \pi) \]

Section 6.5 – Trigonometric Graphs

8. An airplane takes off at a 10° angle and travels at a rate of 250 ft/sec. Approximately how long does it take the plane to reach an altitude of 15,000 ft?

Section 6.5 – Trigonometric Graphs
Chapter 7: Analytic Trigonometry

9. Verify the identity.

\[
\frac{1}{1 - \cos(\gamma)} + \frac{1}{1 + \cos(\gamma)} = 2\csc^2(\gamma)
\]

Section 7.1 – Verifying Trigonometric Identities

10. Verify the identity.

\[
\tan^4(k) - \sec^4(k) = 1 - 2\sec^2(k)
\]

Section 7.1 – Verifying Trigonometric Identities

11. Find all solutions to the equation.

\[
\sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2}
\]

Section 7.2 – Trigonometric Equations

12. Find the solutions that are in the interval \([0, 2\pi]\).

\[
2\tan(t) - \sec^2(t) = 0
\]

Section 7.2 – Trigonometric Equations

13. Approximate, to the nearest 10’, the solutions in the interval \([0^\circ, 360^\circ]\).

\[
\sin^2(t) - 4\sin(t) + 1 = 0
\]

Section 7.2 – Trigonometric Equations
14. If \(\sin(\alpha) = -\frac{4}{5}\) and \(\sec(\beta) = \frac{5}{3}\) for a third-quadrant angle \(\alpha\) and a first-quadrant angle \(\beta\), find the following.
   a. \(\sin(\alpha + \beta)\)
   b. \(\tan(\alpha + \beta)\)
   c. The quadrant containing \(\alpha + \beta\)

Section 7.3 – The Addition and Subtraction Formulas

15. Find the exact values of \(\sin 2\theta\), \(\cos 2\theta\), and \(\tan 2\theta\) given the information below.

\[
\sec \theta = -3 \quad 90^\circ < \theta < 180^\circ
\]

Section 7.4 – Multiple-Angle Formulas

16. Find the solutions that are in the interval \([0, 2\pi)\).

\[
\sin 2t + \sin t = 0
\]

Section 7.4 – Multiple-Angle Formulas

17. Find the exact value.
   a. \(\cot(\sin^{-1} \left(\frac{2}{3}\right))\)
   b. \(\sec(\tan^{-1}(-\frac{3}{5}))\)
   c. \(\csc(\cos^{-1}(-\frac{1}{4}))\)

Section 7.6 – The Inverse Trigonometric Functions

18. Find the exact value whenever it is defined.
   a. \(\sin(\arcsin(\frac{1}{2}) + \arccos(0))\)
   b. \(\cos[\arctan(-\frac{3}{4}) - \arcsin(\frac{4}{5})]\)
   c. \(\tan(\arctan(\frac{4}{3}) + \arccos(\frac{8}{17}))\)

Section 7.6 – The Inverse Trigonometric Functions
19. Find the exact value whenever it is defined.
   a. \( \sin\left[2 \arccos\left(-\frac{3}{5}\right)\right] \)
   b. \( \cos\left[2 \sin^{-1}\left(\frac{15}{17}\right)\right] \)
   c. \( \tan\left[2 \tan^{-1}\left(\frac{3}{4}\right)\right] \)

Section 7.6 – The Inverse Trigonometric Functions

20. Write the expression as an algebraic expression in terms of \( x \) for \( x > 0 \).

\[
\sin\left(2 \sin^{-1} x\right)
\]

Section 7.6 – The Inverse Trigonometric Functions

21. Use inverse trigonometric functions to find the solutions of the equation that are in \([0, 2\pi)\). Approximate to four decimal places.

\[
\cos^2 x + 2 \cos x - 1 = 0
\]

Section 7.6 – The Inverse Trigonometric Functions
Chapter 8: Applications of Trigonometry

22. Solve \( \triangle ABC \).
\[
\gamma = 81^\circ \quad c = 11 \quad b = 12
\]

Section 8.1 – The Law of Sines

23. A forest ranger at an observation point A sights a fire in the direction \( N27^\circ 10' E \). Another ranger at an observation point B, 6 miles east of point A, sights the same fire in the direction \( N52^\circ 40' W \). Approximate the distance between point A and the fire.

Section 8.1 – The Law of Sines

24. Solve \( \triangle ABC \).
\[
a = 25.0 \quad b = 80.0 \quad c = 60.0
\]

Section 8.2 – The Law of Cosines

25. A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.

Section 8.2 – The Law of Cosines

26. Approximate the area of \( \triangle ABC \).
\[
a = 80.1^\circ \quad a = 8.0 \quad b = 3.4
\]

Section 8.2 – The Law of Cosines

27. Approximate the area of \( \triangle ABC \).
\[
a = 25.0 \quad b = 80.0 \quad c = 60.0
\]

Section 8.2 – The Law of Cosines
28. Approximate the areas of the parallelogram that has sides of length \( a \) and \( b \) (in feet) of one angle at the vertex has measure \( \theta \)

\[
a = 12.0 \quad b = 16.0 \quad \theta = 40^\circ
\]

\textit{Section 8.2 – The Law of Cosines}
Chapter 9: Systems of Equations and Inequalities

29. Use the method of substitution to solve the system.

\[
\begin{align*}
  x^2 + y^2 &= 16 \\
  2y - x &= 4
\end{align*}
\]

Section 9.1 – Systems of Equations

30. Use the method of substitution to solve the system.

\[
\begin{align*}
  y^2 + 4x^2 &= 4 \\
  9y^2 + 16x^2 &= 140
\end{align*}
\]

Section 9.1 – Systems of Equations

31. Use the method of substitution to solve the system.

\[
\begin{align*}
  x &= y^2 - 4y + 5 \\
  x - y &= 1
\end{align*}
\]

Section 9.1 – Systems of Equations

32. The price of admission to a high school play was $3.00 for students and $4.50 for nonstudents. If 450 tickets were sold for a total of $1555.50, how many of each kind were purchased?

Section 9.2 – Systems of Linear Equations in Two Variables

33. A small furniture company manufactures sofas and recliners. Each sofa requires 8 hours of labor and $180 in materials, while a recliner can be built for $105 in 6 hours. The company has 340 hours of labor available each week and can afford to buy $6750 worth of materials. How many recliners and sofas can be produced if all labor hours and all materials must be used?

Section 9.2 – Systems of Linear Equations in Two Variables
Chapter 11: Topics from Analytic Geometry

34. Find the vertex, focus, and directrix of the parabola. Sketch its graph, showing the focus and the directrix.

\[ y = x^2 - 4x + 2 \]

Section 11.1 – Parabolas

35. Find an equation of the parabola that satisfies the given conditions.

Focus: \( F(6, 4) \)
Directrix: \( y = -2 \)

Section 11.1 – Parabolas

36. Find the equation of the parabola that satisfies the given conditions.

Vertex \( V(-3, 5) \), axis parallel to the \( x \)-axis, and passing through the point \((5, 9)\).

Section 11.1 – Parabolas

37. Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

\[ \frac{(x - 3)^2}{16} + \frac{(y + 4)^2}{9} = 1 \]

Section 11.2 – Ellipses

38. Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

\[ 4x^2 + 9y^2 - 32x - 36y + 64 = 0 \]

Section 11.2 – Ellipses

39. Find an equation of the ellipse that has its center at the origin and satisfies the given conditions:

Vertices: \( V(\pm 8, 0) \)
Foci: \( F(\pm 5, 0) \)

Section 11.2 – Ellipses
40. Find the vertices, the foci, and the equation of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

\[ 4y^2 - x^2 + 40y - 4x + 60 = 0 \]

*Section 11.3 – Hyperbolas*

41. Find the equation of the hyperbola that has its center at the origin and satisfies the given conditions:

- Vertices: \( V(\pm 4, 0) \)
- Passing through \((8, 2)\)

*Section 11.3 – Hyperbolas*