Math 11000 Exam Jam

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1 Algebra Review

1. Evaluate $-x^2 - 7x$ for $x = -3$.

2. Solve for $x$.

\[ 8x - (4x + 5) = 19 \]

3. Find the slope and the $y$-intercept of the equation. Also graph the line.

\[ 2x + 3y = 6 \]

4. Simplify.

\[ (8x^4 + 7x^3 - 2) - (2x^3 + x^2 - 3) \]

5. Multiply.

\[ (2x - 3)^2 \]

6. Factor completely. If it is prime, state this.

\[ 18t^5 - 12t^4 + 6t^3 \]
2 Functions and Graphs

7. Find the function values.

\[ f(n) = 5n^2 + 4n \]

(a) \( f(-1) \)

(b) \( f(3) \)

(c) \( f(2a) \)

8. For the graph of the function \( f \), determine the domain and range of \( f \) and find \( f(1) \) where \( f(x) = x^2 + 1 \).

9. In 2000, the life expectancy of females born that year was 79.7 years. In 2010, it was 81.1 years. Let \( E(t) \) represent life expectancy and \( t \) the number of years since 2000.

(a) Find a linear function that fits the data.

(b) Use the linear function of part (a) to predict the life expectancy of females in 2020.

10. Let \( F(x) = x^2 - 2 \) and \( G(x) = 5 - x \). Find the following:

(a) \((F + G)(3)\)

(b) \((F \cdot G)(x)\)

11. Let \( f(x) = 16x - 4 \). Find the inverse of the function.
3 Exponents and Radicals

12. Simplify. Variables may represent any real number, so remember to use absolute-value notation when necessary. If a root cannot be simplified, state this.

\[ \sqrt{y^2 + 16y + 64} \]

13. Use rational exponents to simplify. Write the answer using radical notation.

\[ 12\sqrt{a^6} \]

14. Simplify. Assume that no radicands were formed by raising negative numbers to even powers.

(a) \( \sqrt{45} \)
(b) \( \sqrt{120} \)
(c) \( \sqrt{6}\sqrt{33} \)

15. Rationalize the denominator.

\[ \frac{\sqrt{5}}{\sqrt{8}} \]
4 Quadratic Functions and Equations

16. Solve by factoring and using the principal of zero products.
   (a) \( x^2 + 4x - 21 = 0 \)
   (b) \( 64 + x^2 = 16x \)
   (c) \( 4t^2 = 8t \)

17. Solve for \( x \).
   \( 4x^2 - 12 = 0 \)

18. Solve. (Find all complex-number solutions.)
   \( (t + 5)^2 = 12 \)

19. Let \( f(x) = 6x^2 - 7x - 20 \). Find \( x \) such that \( f(x) = 0 \).

20. A number is 6 less than its square. Find all such numbers.

21. The distance an object travels in a straight line is given by the function \( s(t) = t^2 - 8t \), where \( s \) is in feet and \( t \) is the number of seconds the object has been in motion. How long does it take the object to move 9 feet?

22. Graph the function and find the vertex, the axis of symmetry, and the maximum value or the minimum value.
   \( h(x) = -2(x - 1)^2 - 3 \)

23. Find any \( x \)-intercepts and the \( y \)-intercept. If no intercepts exist, state this.
   \( f(x) = x^2 - 6x + 3 \)

24. Find the vertex.
   \( f(x) = 3x^2 - 12x + 8 \)

25. Sweet Harmony Crafts has determined that when \( x \) hundred dulcimers are built, the average cost per dulcimer can be estimated by
   \[ C(x) = 0.1x^2 - 0.7x + 2.425 \]
   where \( C(x) \) is in hundreds of dollars. What is the minimum average cost per dulcimer and how many dulcimers should be built in order to achieve that minimum?
5 Exponential and Logarithmic Functions

26. Given $f(x) = 5x + 1$ and $g(x) = x^2$, find:
   (a) $(f \circ g)(2)$
   (b) $(g \circ f)(x)$

27. Graph.
   (a) $f(x) = 3^x$
   (b) $g(x) = \left(\frac{1}{4}\right)^x$

28. Solve.

   $\log_2 32 = x$

29. Express as an equivalent expression, using the individual logarithms of $x$, $y$, and $z$.

   $\log_a \frac{x^5}{y^3z}$

30. Use a calculator to find each of the following to four decimal places.
   (a) $\log 7$
   (b) $\ln 9$
   (c) $e^{2.71}$

31. Solve for $x$. Approximate to three decimal places if necessary.
   (a) $4^{x+1} = 16$
   (b) $3^{2x} = 2$
   (c) $10^{x-3} = 5$
   (d) $6e^{0.05x} = 18$

32. Suppose that $P_0$ is invested in a savings account where interest is compounded continuously at 3% per year.
   (a) Express $P(t)$ in terms of $P_0$ and 0.03.
   (b) Suppose that $5000$ is invested. What is the balance after 1 year? After 2 years?
   (c) When will an investment of $5000$ double itself?
6 Systems of Linear Equations

33. Solve the system graphically. Be sure to check your solution. If a system has an infinite number of solutions, use set-builder notation to write the solution set. If a system has no solution, state this.

\[
\begin{align*}
y - x &= 5 \\
2x - 2y &= 10
\end{align*}
\]

34. Solve using the substitution method.

\[
\begin{align*}
3s - 4t &= 14 \\
5s + t &= 8
\end{align*}
\]

35. Ellen wishes to mix candy worth $1.80 per pound with candy worth $2.40 per pound to form 48 pounds of a mixture worth $2.00 per pound. How many pounds of the more expensive candy should she use?

36. Solve each system. If a system’s equations are dependent or if there is no solution, state this.

\[
\begin{align*}
x - y - z &= 1 \\
2x + y + 2z &= 4 \\
x + y + 3z &= 5
\end{align*}
\]

37. The sum of three numbers is 85. The second is 7 more than the first. The third is 2 more than four times the second. Find the numbers.

38. For the following pair of total-cost and total-revenue functions, find the total-profit function and the break-even point.

\[
\begin{align*}
C(x) &= 15x + 3100, \\
R(x) &= 40x
\end{align*}
\]
7 Inequalities

39. Solve algebraically.

\[ 5(t - 3) + 4t < 2(7 + 2t) \]

40. Find the indicated intersection.

\[ \{2, 4, 16\} \cap \{4, 16, 256\} \]

41. Solve and graph the solution set, where \( f(t) = 5t + 3 \).

\[ f(t) < -7 \text{ or } f(t) > 8 \]

42. Solve and graph \( 4x - 1 < 7 \) and \( 1 - 3x \leq -5 \).

43. Graph.

\[ x + y \leq 6, \quad x - y \leq 4 \]

44. Maximize \( F = 6x + 7y \) subject to

\[ 2x + 3y \leq 12 \]
\[ 2x + y \leq 8 \]
\[ x \geq 0 \]
\[ y \geq 0 \]
8 Logic and Truth Tables

45. Let \( p \) represent the statement “She has green eyes” and let \( q \) represent the statement “He is 60 years old.” Translate the symbolic compound statement into words.

\[ \sim p \lor \sim q \]

46. Construct a truth table for the compound statement.

\[ (q \lor \sim p) \lor \sim q \]

47. Construct a truth table for the statement. Identify whether or not it is a tautology.

\[ \sim q \rightarrow p \]

48. For the given conditional statement, write (a) the converse, (b) the inverse, and (c) the contrapositive in “if . . . then” form.

\[ p \rightarrow \sim q \]

49. Negate the statement: Not all people like football.

50. Let \( p \) represent the statement “Today is Saturday” and let \( q \) represent the statement “I will go to the movies.” Translate the symbolic compound statement into words.

\[ \sim p \lor q, \sim (p \land q), p \rightarrow q, \text{ and } \sim p \leftrightarrow q \]

51. Use DeMorgan’s Laws to negate the statement: It is Saturday and it is not raining.

52. Write the contrapositive, converse, and inverse of the conditional statement: If I were young, I would be happy.