Math 15400 Exam Jam

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Chapter 6: The Trigonometric Functions

1. Given \( s = 7 \) cm and \( r = 4 \) cm, answer the following
   a. Find the radian and degree measures of the central angle \( \theta \) subtended by the
      given arc of length \( s \) on a circle of radius \( r \).
   b. Find the area of the sector determined by \( \theta \).

   Section 6.1 – Angles

2. Given a radius 5 in. and 40 rpm, answer the following:
   a. Find the angular speed (in radians per minute).
   b. Find the linear speed of a point on the circumference (in feet per minute).

   Section 6.1 – Angles

3. Use the fundamental identities to write \( \cot \theta \) in terms of \( \sin \theta \) for any acute angle \( \theta \).

   Section 6.2 – Trigonometric Functions of Angles

4. Verify the identity.

   \[
   \sec(\theta) - \cos(\theta) = \tan(\theta) \sin(\theta)
   \]

   Section 6.2 – Trigonometric Functions of Angles

5. Find the exact value.
   a. \( \csc \left( \frac{3\pi}{4} \right) \)
   b. \( \csc \left( -\frac{2\pi}{3} \right) \)

   Section 6.4 – Values of Trigonometric Functions

6. Approximate, to the nearest 0.01 radian, all angles \( \theta \) in the interval \([0,2\pi]\) that satisfy
   the equation.
   a. \( \sin(\theta) = 0.4195 \)
   b. \( \tan(\theta) = -3.2504 \)
   c. \( \sec(\theta) = 1.7452 \)

   Section 6.4 – Values of Trigonometric Functions

7. Find the amplitude, period, and phase shift. Then sketch the graph.

   \[
   y = -2 \sin(3x - \pi)
   \]

   Section 6.5 – Trigonometric Graphs

8. An airplane takes off at a 10° angle and travels at a rate of 250 ft/sec. Approximately
   how long does it take the plane to reach an altitude of 15,000 ft?
Section 6.5 – Trigonometric Graphs
Chapter 7: Analytic Trigonometry

9. Verify the identity.
\[
\frac{1}{1 - \cos(y)} + \frac{1}{1 + \cos(y)} = 2 \csc^2(y)
\]

Section 7.1 – Verifying Trigonometric Identities

10. Verify the identity.
\[
\tan^4(k) - \sec^4(k) = 1 - 2 \sec^2(k)
\]

Section 7.1 – Verifying Trigonometric Identities

11. Find all solutions to the equation.
\[
\sin \left(2x - \frac{\pi}{3}\right) = \frac{1}{2}
\]

Section 7.2 – Trigonometric Equations

12. Find the solutions that are in the interval \([0, 2\pi]\).
\[
2 \tan(t) - \sec^2(t) = 0
\]

Section 7.2 – Trigonometric Equations

13. Approximate, to the nearest 10′, the solutions in the interval \([0°, 360°]\).
\[
\sin^2(t) - 4 \sin(t) + 1 = 0
\]

Section 7.2 – Trigonometric Equations
14. If \( \sin(\alpha) = -\frac{4}{5} \) and \( \sec(\beta) = \frac{5}{3} \) for a third-quadrant angle \( \alpha \) and a first-quadrant angle \( \beta \), find the following.
   a. \( \sin(\alpha + \beta) \)
   b. \( \tan(\alpha + \beta) \)
   c. The quadrant containing \( \alpha + \beta \)

15. Find the exact values of \( \sin 2\theta \), \( \cos 2\theta \), and \( \tan 2\theta \) given the information below.

   \[ \sec \theta = -3 \quad 90^\circ < \theta < 180^\circ \]

16. Find the solutions that are in the interval \([0, 2\pi)\).

   \[ \sin 2t + \sin t = 0 \]

17. Find the exact value.
   a. \( \cot(\sin^{-1}\left(\frac{2}{3}\right)) \)
   b. \( \sec(\tan^{-1}\left(-\frac{3}{5}\right)) \)
   c. \( \csc(\cos^{-1}\left(-\frac{1}{4}\right)) \)

18. Find the exact value whenever it is defined.
   a. \( \sin\left(\arcsin\left(\frac{1}{2}\right) + \arccos(0)\right) \)
   b. \( \cos[\arctan\left(-\frac{3}{4}\right) - \arcsin\left(\frac{4}{5}\right)] \)
   c. \( \tan\left(\arctan\left(\frac{4}{3}\right) + \arccos\left(\frac{8}{17}\right)\right) \)
19. Find the exact value whenever it is defined.
   a. \( \sin \left[ 2 \arccos \left( -\frac{3}{5} \right) \right] \)
   b. \( \cos \left[ 2 \sin^{-1} \left( \frac{15}{17} \right) \right] \)
   c. \( \tan \left[ 2 \tan^{-1} \left( \frac{3}{4} \right) \right] \)

Section 7.6 – The Inverse Trigonometric Functions

20. Write the expression as an algebraic expression in terms of \( x \) for \( x > 0 \).

\[
\sin \left( 2 \sin^{-1} x \right)
\]

Section 7.6 – The Inverse Trigonometric Functions

21. Use inverse trigonometric functions to find the solutions of the equation that are in \([0, 2\pi)\). Approximate to four decimal places.

\[
\cos^2 x + 2 \cos x - 1 = 0
\]

Section 7.6 – The Inverse Trigonometric Functions
Chapter 8: Applications of Trigonometry

22. Solve \( \Delta ABC \).
   \[
   \gamma = 81^\circ \quad c = 11 \quad b = 12
   \]
   \textit{Section 8.1 – The Law of Sines}

23. A forest ranger at an observation point A sights a fire in the direction \( N27^\circ10' E \). Another ranger at an observation point B, 6 miles east of point A, sights the same fire in the direction \( N52^\circ40' W \). Approximate the distance between point A and the fire.
   \textit{Section 8.1 – The Law of Sines}

24. Solve \( \Delta ABC \).
   \[
   a = 25.0 \quad b = 80.0 \quad c = 60.0
   \]
   \textit{Section 8.2 – The Law of Cosines}

25. A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.
   \textit{Section 8.2 – The Law of Cosines}

26. Approximate the area of \( \Delta ABC \).
   \[
   a = 80.1^\circ \quad a = 8.0 \quad b = 3.4
   \]
   \textit{Section 8.2 – The Law of Cosines}

27. Approximate the area of \( \Delta ABC \).
   \[
   a = 25.0 \quad b = 80.0 \quad c = 60.0
   \]
   \textit{Section 8.2 – The Law of Cosines}
28. Approximate the areas of the parallelogram that has sides of length \( a \) and \( b \) (n feet) of one angle at the vertex has measure \( \theta \)

\[
\begin{align*}
a &= 12.0 \\
b &= 16.0 \\
\theta &= 40^\circ
\end{align*}
\]

*Section 8.2 – The Law of Cosines*
Chapter 9: Systems of Equations and Inequalities

29. Use the method of substitution to solve the system.

\[
\begin{align*}
\begin{cases}
  x^2 + y^2 &= 16 \\
  2y - x &= 4
\end{cases}
\end{align*}
\]

Section 9.1 – Systems of Equations

30. Use the method of substitution to solve the system.

\[
\begin{align*}
\begin{cases}
  y^2 + 4x^2 &= 4 \\
  9y^2 + 16x^2 &= 140
\end{cases}
\end{align*}
\]

Section 9.1 – Systems of Equations

31. Use the method of substitution to solve the system.

\[
\begin{align*}
\begin{cases}
  x &= y^2 - 4y + 5 \\
  x - y &= 1
\end{cases}
\end{align*}
\]

Section 9.1 – Systems of Equations

32. The price of admission to a high school play was $3.00 for students and $4.50 for nonstudents. If 450 tickets were sold for a total of $1555.50, how many of each kind were purchased?

Section 9.2 – Systems of Linear Equations in Two Variables

33. A small furniture company manufactures sofas and recliners. Each sofa requires 8 hours of labor and $180 in materials, while a recliner can be built for $105 in 6 hours. The company has 340 hours of labor available each week and can afford to buy $6750 worth of materials. How many recliners and sofas can be produced if all labor hours and all materials must be used?

Section 9.2 – Systems of Linear Equations in Two Variables

Chapter 11: Topics from Analytic Geometry

34. Find the vertex, focus, and directrix of the parabola. Sketch its graph, showing the focus and the directrix.

\[
y = x^2 - 4x + 2
\]

Section 11.1 – Parabolas
35. Find an equation of the parabola that satisfies the given conditions.

Focus: \( F(6,4) \)
Directrix: \( y = -2 \)

Section 11.1 – Parabolas

36. Find the equation of the parabola that satisfies the given conditions.

Vertex \( V(-3,5) \), axis parallel to the x-axis, and passing through the point \((5,9)\).

Section 11.1 – Parabolas

37. Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

\[
\frac{(x - 3)^2}{16} + \frac{(y + 4)^2}{9} = 1
\]

Section 11.2 – Ellipses

38. Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

\[
4x^2 + 9y^2 - 32x - 36y + 64 = 0
\]

Section 11.2 – Ellipses

39. Find an equation of the ellipse that has its center at the origin and satisfies the given conditions:

Vertices: \( V(\pm8,0) \)
Foci: \( F(\pm5,0) \)

Section 11.2 – Ellipses

40. Find the vertices, the foci, and the equation of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

\[
4y^2 - x^2 + 40y - 4x + 60 = 0
\]

Section 11.3 – Hyperbolas

41. Find the equation of the hyperbola that has its center at the origin and satisfies the given conditions:

Vertices: \( V(\pm4,0) \)
Passing through \((8,2)\)

Section 11.3 – Hyperbolas
Answers

1. 
   a. $\theta = \frac{7}{4}$
   b. $A = 14$ cm$^2$

2. 
   a. $80\pi \text{ rad min}^{-1}$
   b. $\frac{100\pi}{3} \text{ ft min}^{-1}$

3. $\cot(\theta) = \frac{\sqrt{1 - \sin^2(\theta)}}{\sin(\theta)}$

4. $\tan(\theta) \sin(\theta) = \tan(\theta) \sin(\theta)$

5. 
   a. $\sqrt{2}$
   b. $-\frac{2\sqrt{3}}{3}$

6. 
   a. $\theta_1 = 0.43, \theta_2 = 2.71$
   b. $\theta_1 = 1.87, \theta_2 = 5.01$
   c. $\theta_1 = 0.96, \theta_2 = 5.32$

7. Amplitude: $|2| = 2$
   Period: $\frac{2\pi}{|3|} = \frac{2\pi}{3}$
   Phase Shift: $-\left(-\frac{\pi}{3}\right) = \frac{\pi}{3}$

8. 346 seconds

9. $\frac{2}{\sin^2(\gamma)} = 2 \csc^2(\gamma)$

10. $1 - 2 \sec^2(k) = 1 - 2 \sec^2(k)$

11. $x_1 = \frac{\pi}{4} + \pi n, x_2 = \frac{7\pi}{12} + \pi n$

12. $t = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

13. $t = \{15^\circ30', 164^\circ30'\}$
14. 
   a. $-\frac{24}{25}$
   b. $-\frac{24}{7}$
   c. Fourth Quadrant

15. $\sin 2\theta = -\frac{4\sqrt{2}}{9}, \cos 2\theta = -\frac{7}{9}, \tan 2\theta = \frac{4\sqrt{2}}{7}$

16. $t = \{0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

17. 
   a. $\frac{\sqrt{5}}{2}$
   b. $\frac{\sqrt{34}}{5}$
   c. $\frac{4\sqrt{15}}{15}$

18. 
   a. $\frac{\sqrt{3}}{2}$
   b. 0
   c. $-\frac{77}{36}$

19. 
   a. $-\frac{24}{25}$
   b. $-\frac{161}{289}$
   c. $\frac{24}{7}$

20. $2x\sqrt{1 - x^2}$

21. 5.1395

22. No solution

23. $x \approx 3.70$ miles

24. $\alpha = 12.43^\circ, \beta = 43.53^\circ, \gamma = 124.04^\circ$

25. 24.98°
26. \( A \approx 13.15 \text{ units}^2 \)

27. \( A \approx 516.56 \text{ units}^2 \)

28. \( A \approx 123.4 \text{ ft}^2 \)

29. \((-4,0) \text{ and } \left(\frac{12}{5}, \frac{16}{5}\right)\)

30. No solutions

31. \((2,1) \text{ and } (5,4)\)

32. 313 student tickets, 137 non-student tickets

33. 20 sofas, 30 recliners

34. Vertex: \(V(2, -2)\), Focus: \(F(2, -\frac{7}{4})\), Directrix: \(y = -\frac{9}{4}\)

35. \(12(y - 1) = (x - 6)^2\)

36. \((y - 5)^2 = 2(x + 3)\)

37. \(V(3 \pm 4, -4), F(3 \pm \sqrt{7}, -4)\)

38. \(C(4,2), V(4 \pm 3, 2), F(4 \pm \sqrt{5}, 2)\)

39. \(\frac{x^2}{64} + \frac{y^2}{39} = 1\)

40. \(C(-2, -5), V(-2, -5 \pm 3), F(-2, -5 \pm 3\sqrt{5})\)

Asymptotes: \(y = \frac{x}{2} - 4\) and \(y = -\frac{x}{2} - 6\)

41. \(\frac{x^2}{16} - \frac{3y^2}{4} = 1\)