M118 Exam Jam Concise

Contents

Chapter 2: Set Theory

1. Let $U = \{a, b, c, d, e, f\}$, $A = \{a, e\}$, $B = \{b, c, d, e\}$, and $C = \{c, f\}$. Find the following.
   (a) $(A \cap B) \cup C$
   (b) $A \cap (B \cup C)$
   (c) $A \times B'$

2. Of the 72 students in a mathematics class, 37 are taking French, 32 are taking German, and 9 are taking both French and German. How many students are taking neither French nor German?

3. A sample of 100 students was surveyed about their preferences for types of music played on the radio. The choices to select from were rock, blues, and country. The results of the survey are listed below:

72 like rock	22 like rock and blues	27 like country and blues
40 like blues	20 like rock and country	12 like all three
37 like country		

   (a) How many students surveyed like only one of the three types of music?

   (b) How many students surveyed like none of the three types of music?

   (c) How many students surveyed like at least two of the three types of music?

4. Let A and B be subsets of a universal set U, where:

$$\begin{aligned} n(U) &= 40 \\ n(A' \cap B') &= 8 \\ n(B - A) &= 10 \\ n(A - B) &= 10 \end{aligned}$$

Find $n(A \cap B)$

Chapter 3: Combinatorics

5. How many license plates can be produced using five symbols on each plate where the first two symbols are different letters of the alphabet and the following three symbols are different numbers selected from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$? No number or letter can be repeated on the same license plate.

6. How many distinguishable arrangements can be made from all the letters in the word MATHEMATICS?

7. How many committees of four can be formed from 20 Republicans and 15 Democrats if
   (a) two members of each party must be on the committee?
   (b) at least one Republican and at least one Democrat must be on the committee?

8. How many ways can 6 different contracts be distributed amongst 10 different firms, if any one firm can be awarded multiple contracts?

9. How many different 3-digit EVEN numbers can be formed from the set$\{1,2,3,4,5,6,7,8\}$ if it cannot start with a 1 or 2 and repetition is allowed?

Chapter 4: Probability

10. Let A and B be disjoint events with $\Pr[A] = 0.55$ and $\Pr[B] = 0.25$. Find $\Pr[A \cup B]$.

11. Let C and D be events such that $\Pr[C] = 0.8$, $\Pr[D] = 0.4$, and $\Pr[C \cup D] = 0.9$.
    (a) Evaluate $\Pr[D|C]$.
    (b) Evaluate $\Pr[C|D]$.

12. Five cards are drawn at random from a standard deck of 52. What is the probability that all five cards are of the same suit?

13. It is believed that 8% of the population has hepatitis. A medical firm has a new test to detect hepatitis. It was found that if a person has hepatitis, the test will detect it (show a positive result) in 96% of the cases; it was also found that it will show a positive result in 3% of those who do not have hepatitis. What is the probability that a person has hepatitis given that they tested positive?

14. An unfair coin with $\Pr[H] = 0.25$ is flipped. If the flip results in a head, a student is selected at random from a class of ten boys and twelve girls. Otherwise, a student from a different class containing six boys and five girls is selected.

    (a) What is the probability of selecting a girl?

    (b) What is the probability of selecting a girl given that the flip resulted in heads?

    (c) What is the probability of flipping heads given that a girl was selected?

15. A basketball player makes free throws with a 0.8 probability. What is the probability that the player will make at least five of the next six free throws?

16. Given that the probability of a male child is 0.52, what is the probability that a family with four children will have at least one male child?

17. A box contains five red and two blue marbles. If two marbles are drawn from the box without replacement, what is the probability that both marbles are the same color?

Chapter 5: Statistics

18. Two coins are randomly selected from a pocket containing two quarters, one dime, and one nickel. A random variable X is defined as the value of coins (in cents). Find the expected value.

19. A teacher says that the top 4% of the class received an A on the last test. The scores were normally distributed with mean 67 and standard deviation 7. Find the minimum score required to get an A.

20. The mean length of angelfish (a popular tropical fish for aquariums) is 10.2 cm with a standard deviation of 2.1 cm. Assuming that the length of angelfish is a normal random variable, find the percent of tropical fish between 7 and 13 cm in length.

21. Suppose 10% of all people are left-handed. If 300 people are selected at random, find the expected number of left-handed people in the sample. What is the standard deviation?

22. Find the standard deviation for the probability density function with $E[X] = 2.0$.

Value of X	Probability
0	0.4
2	0.3
4	0.2
6	0.1

23. A carnival game costs \$1 to play. The probability that you win \$50 is 0.001. The probability that you win \$10 is 0.05. The probability that you win \$5 is 0.10. What is your expected return per game?

Chapter 6: Linear Equations and Matrix Algebra

24. Solve the following system of linear equations using the substitution or elimination methods.

$$\begin{aligned} 3x + 4y &= 2 \\ 6x - 8y &= 0 \end{aligned}$$

25. Perform the indicated operation given the following matrices:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 6 \\ 1 & -2 & 0 \\ 0 & 1 & -3 \end{bmatrix}$$

(a) $B - 3A$
(b) AB

26. Solve the following system of linear equations using the substitution or elimination methods.

$$\begin{aligned} x + y + z &= 6 \\ 2x - y - z &= -3 \\ x - 2y + 3z &= 6 \end{aligned}$$

27. Find AB and BA, if possible:

$$A = \begin{bmatrix} -4 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \\ 6 & -9 \end{bmatrix}$$

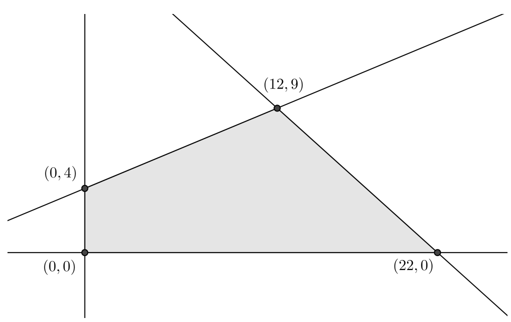
28. A large state university offers two math courses: Finite Math and Applied Calculus. Each section of Finite Math has 60 students and each section of Applied Calculus has 50. The department will offer a total of 110 sections in a semester, and 6000 students would like to take a math course. How many sections of each course should the department offer in order to fill all sections and accomodate all of the students? Hint: Set up a system of equations where $x =$ the number of Finite sections and $y =$ the number of Applied Calculus sections.

29. Solve the following system of linear equations using the substitution or elimination methods.

$$\begin{aligned} x + 4y &= 12 \\ -3x - 12y &= -24 \end{aligned}$$

Chapter 7: Linear Programming: Graphical Solutions

30. Find the maximum value of $F = -30x + 50y$ in the feasible region shown below.



31. Graph the following system of linear inequalities and shade the feasible region. Is it bounded or unbounded? Find the coordinates of all corner points. Show all your work.

$$\begin{aligned} 2x + 4y &\geq 8 \\ 4x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

32. A farm consists of 240 acres of land. The farmer wishes to plant this acreage in corn or oats. Profit per acre in corn is \$40 and that in oats is \$30. An additional restriction is that the total hours of labor during the season is 320. Each acre of land in corn requires 2 hours of labor, whereas oats require 1 hour per acre. Determine how the land should be divided between corn and oats in order to give maximum profit.

Chapter 9: Markov Chains

33. For the given transition matrix and state vector, find the state vector

$$T = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \qquad P_0 = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}$$

(a) one transition later.

(b) two transitions later.

34. A town has two plumbers. 50% of those who call plumber A will call plumber B the next time, and 40% of those who call plumber B will call plumber A the next time. If plumber A was most recently used for a repair, what is the probability that plumber B will be used two repairs later?

35. For the given transition matrix and state vector, find the state vector

$$T = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad P_0 = \begin{bmatrix} 0.1 & 0.4 & 0.5 \end{bmatrix}$$

(a) one transition later.

(b) two transitions later.

36. Aaron sends his mother a birthday card every year. Sometimes he sends a funny card. Sometimes he sends a sentimental card. His mother has kept track of the type sent and calculates that a funny card is followed the next year by a sentimental one 80% of the time. A sentimental card is followed by a funny one 60% of the time. If a sentimental card is sent this year, what is the probability that another sentimental card will be sent four years from now?

37. Taylor's and Sower's are two stores that do alterations of garments. 40% of Taylor's customers take their next garment to Sower's, and 30% of Sower's customers take their next job to Taylor's. In the long run, what will be the distribution of the market between these two stores? (i.e., find the steady state vector)

38. Determine whether the following transition matrix is regular.

$$\begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.2 & 0 & 0.8 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

39. Determine whether the following transition matrix is regular.

$$\begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$