Contents

1 Matrices

1.1 Matrix Operations

Given the matrices

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} -1 & 1 & 0 \\ 4 & -2 & 2 \end{pmatrix}$$

Compute the following, if they are defined.

1. $\mathbf{AB}$
2. $\mathbf{BA}$
3. $\mathbf{A} + \mathbf{B}$
4. $3\mathbf{A}$

1.2 Systems of Equations

Find all solutions, if any, to the given system of equations using either Gaussian elimination or Gauss-Jordan elimination.

$$4x + 3y = 5$$
$$3x - 2y = 8$$

Find all solutions to the following system of equations.

$$\begin{aligned} x + 2z &= 1 \\ x + y + z &= 3 \\ 2x + 3y + 2z &= 9 \end{aligned}$$

1.3 Determinants and Inverses

Find the determinants of the following matrices **A** and **B**

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -1 & -2 \\ -3 & 3 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 & 1 & 0 \\ 3 & 2 & 1 \\ 0 & 4 & 2 \end{pmatrix}$$

Find the inverses of the following matrices.

$$\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$$

1.4 Eigenvalues and Eigenvectors

Compute the eigenvalues and eigenvectors for the following matrix.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

1.5 Difference Equations

Solve the following initial value difference equation. (Note: Not all classes cover this material.)

$$x_{n+1} - 5x_n + 6x_{n-1} = 0, \quad x_0 = 4, \quad x_1 = 9$$

2 Functions of Several Variables

2.1 Function Values

The surface area of a person (in m^2) can be approximated using a formula known as the Haycock formula. The formula is a function of two variables, the person's height in centimeters and the person's weight in kilograms. The formula is as follows

$$\mathrm{S}(h, w) = .024265h^{.3964}w^{.5378}$$

Estimate the surface area of a person who is 165cm tall and weighs 80kg.

2.2 Partial Derivatives

Find first and second order partial derivatives of the function $f(x, y) = 3xy^2 + 2xy + x^2$.

2.3 Maximum-Minimum Value Problems

Find all relative maximum and minimum values of the function

$$f(x, y) = 4xy - x^3 - y^2$$

and verify your result using the D-Test.

2.4 Separable Differential Equations

Find the general solution to the given separable differential equations

1. $\dfrac{dy}{dx} = y \tan x$
2. $\dfrac{1}{\sin x} \dfrac{dy}{dx} = y \cos x$
3. $\dfrac{1}{(\sin x + \cos x)^2} \dfrac{dy}{dx} = y^2$

3 First Order Differential Equations

3.1 First Order Linear Differential Equations

Find the general solution to the given differential equations

1. $y' + 2y = e^t$
2. $y' + \frac{2}{x}y = \frac{2e^{x^2}}{x}$
3. $y' + y\tan x = \cos x$

Solve the following the initial value problem.

$$\frac{dy}{dx} = x - y,\ y(0) = 2$$

3.2 Approximating Solutions

Let $y(x)$ be a solution to the differential equation

$$y' = x^2 y$$

with initial condition $y(1) = 1$. Approximate $y(1.5)$ using Euler's Method with $\Delta x = .1$

3.3 Autonomous Differential Equations and Stability

In a confined population, $y(t)$ is the number of people who have contracted a contagious but nonfatal disease after t days. Then $y(t)$ is modeled by the autonomous differential equation

$$y' = ky(P - y)$$

1. Determine the equilibrium solutions to the differential equation
2. Assess the stability of each equilibrium solution
3. Assuming that only one or two people have the disease initially, what happens to the population in the long run?

4 Higher Order and Systems of Differential Equations

4.1 Higher Order Homogeneous Differential Equations

Solve for the general solution of the given homogeneous differential equations.

1. $y''' + y'' - 6y' = 0$
2. $y'' + 4y' + 4y = 0$
3. $y'' + 2y' + 2y = 0$

4.2 Higher Order Non-homogeneous Differential Equations

Find the solution to the given differential equation

$$y'' + 4y = 16x$$

with initial conditions $y(0) = 2$ and $y'(0) = -3$

4.3 Reduction Methods for Systems of Differential Equations

Use the reduction method to find the general solution of the systems of differential equations

$$\begin{aligned} x' &= y + 2t + 3 \\ y' &= -x + 4t - 2 \end{aligned}$$

4.4 Matrix Methods for Systems of Differential Equations

Use matrix methods to find the general solution and the stability of the origin for the system of differential equations

$$x' = -x + y$$
$$y' = 2y$$

4.5 Special Cases in Matrix Methods for Systems of Differential Equations

Find the general solution to the given systems of differential equations.

1. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
2. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

4.6 Applications of Differential Equations

On a wildlife preserve, the population of gorillas is modeled by the differential equation

$$\frac{dy}{dt} = .1y(100 - y)$$

Where t is measured in years.

1. Find the equilibrium solutions to the differential equation
2. Compute the general solution to the differential equation
3. Given that the preserve had 45 gorillas 4 years ago, find the number of gorillas currently on the preserve
4. Describe what happens to the gorilla population in the long run