Contents

1 Limits

Evaluate the following limits.

1. $\lim_{x\to 3} \dfrac{2x-6}{x^2-9}$
2. $\lim_{x\to 0} \dfrac{x+x^2}{\sqrt{x}}$
3. $\lim_{x\to 0} \dfrac{\sin(6x)}{x}$
4. $\lim_{x\to 4} \dfrac{x^2-16}{x^2+4}$

2 Derivatives

2.1 Difference Quotients

For the following function, (a) formulate the difference quotient in terms of x and h, (b) simplify the difference quotient, (c) take the limit as $h \to 0$ to find the derivative.

$$f(x) = \frac{1}{\sqrt{x}}$$

2.2 Average Rate of Change

Find the average rate of change of the function

$$f(x) = \sqrt{x+2}$$

as x changes from 2 to 7.

2.3 Derivative Rules

Evaluate the derivatives of the following functions.

1. $f(x) = \dfrac{3}{x} + \dfrac{x}{3}$
2. $f(x) = 3(x^2+x)^3$
3. $y = \ln\left(\dfrac{x^2}{1+x^2}\right)$
4. $y = e^{\sin x}$
5. $q(v) = \cos(\ln v)$

2.4 Tangent Lines

Find the equation of the tangent line to the function

$$f(x) = (x-1)e^x + 1$$

at $x = 1$.

2.5 Curve Sketching

Consider the function

$$y = x^3 - 9x^2 + 24x - 10$$

(a) Find the critical points of this function and determine if these points are points of relative maxima or relative minima. (b) Find the intervals where the function is increasing or decreasing. (c) Find the inflection points of this function. (d) Find the intervals where the graph of this function is concave up and concave down. (e) Sketch the graph of this function.
Consider the function

$$y = \frac{x^2 + 1}{x^2 + 4}$$

(a) Find the critical points of this function and determine if these points are points of relative maxima or relative minima. (b) Find the intervals where the function is increasing or decreasing. (c) Find the inflection points of this function. (d) Find the intervals where the graph of this function is concave up and concave down. (e) Sketch the graph of this function.

3 Applications of Derivatives

3.1 Instantaneous Rates of Change

Given a ball whose position, in feet, above the ground is given by $y = 50t - 16t^2$, where t is the time, find

1. how far off the ground is the ball when $t = 1$.
2. the average velocity from $t = 0$ to $t = 2$.
3. the instantaneous velocity at $t = 1$.

3.2 Optimization

Find two positive numbers whose product is 36 and whose sum is a minimum.
A drug company is designing an open-top rectangular box with a square base, that will hold 32000 cubic centimeters. Determine the dimensions x and y that will yield the minimum surface area.
An open-top box is to be constructed from a 8×15 inch piece of cardboard by cutting out the same size square from each of the four corners and then folding up the resulting sides. Find the largest possible volume of such a box.

3.3 Related Rates

A ladder 25 ft long leans against a vertical wall. If the lower end is being moved away from the wall at a rate of 6 ft/sec, how fast is the height of the top decreasing when the lower end is 7 ft from the wall?

4 Integrals

4.1 Indefinite Integrals

Evaluate the following integrals.

1. $\displaystyle\int \frac{\ln x}{x}\,dx$
2. $\displaystyle\int x(x-6)^3\,dx$

3. $\int \tan x \, dx$
4. $\int \dfrac{dx}{x \ln x}$
5. $\int xe^{-x} \, dx$

4.2 Definite Integrals

Evaluate the following definite integrals.

1. $\int_0^3 5x \ln x \, dx$
2. $\int_1^2 e^{2x} \, dx$
3. $\int_0^\pi \sin x \, dx$

4.3 Areas between Functions

Find the area bounded by $y = \sin x$ and $y = \cos x$ when $0 \leq x \leq \pi/4$.
Set up but do not evaluate the integral that expresses the area bounded by $y = 1 - x^2$ and $y = -x$.

4.4 Volumes of Revolution

Look at the region bounded by $y = x$ and $y = x^2$ from $x = 0$ to $x = 1$.

1. Calculate the volume generated from rotating this region about the x-axis.
2. Calculate the volume generated from rotating this region about the y-axis.

Find the volume of the solid generated by revolving the region bounded by $y = 2x^2$, $x = 1$, and the x-axis about the line $x = 2$.

4.5 Improper Integrals

Evaluate the following improper integrals.

1. $\int_{-\infty}^0 xe^x \, dx$
2. $\int_e^\infty \dfrac{\ln(x)}{x} \, dx$
3. $\int_0^\infty 2xe^{-x^2} \, dx$