Contents

1 Differentiation **2**
1.1 Review from Math 22100 2
1.2 L'Hospital's Rule 3
1.3 Applications of Derivatives 4
1.4 Newton's Method 5

2 Integration **6**
2.1 General Power Rule Integrals 6
2.2 Logarithmic and Exponential Integrals 7
2.3 Trigonometric Integrals 8
2.4 Inverse Trigonometric Forms 10
2.5 Trigonometric Substitution 11
2.6 Integration by Parts 12
2.7 Integration of Rational Functions 13

3 Series **14**
3.1 Geometric Series 14
3.2 Tests for Convergence 15
3.3 Maclaurin Series 16
3.4 Operations with Series 17
3.5 Computations with Series 18
3.6 Fourier Series 19

4 First-Order Differential Equations **20**
4.1 Solutions to Differential Equations 20
4.2 Separation of Variables 21
4.3 First-Order Linear Differential Equations 22
4.4 Applications of Differential Equations 23

5 Higher Order Differential Equations **24**
5.1 Higher-Order Homogeneous Differential Equations 24
5.2 Auxiliary Equations 25
5.3 Non-homogeneous Differential Equations 26
5.4 Applications of Second-Order Equations 27
5.5 Computing the Laplace Transformation 28
5.6 Computing the Inverse Laplace Transformation 29
5.7 Solving Differential Equations Using Laplace Transformations 31

1 Differentiation

1.1 Review from Math 22100

Find the derivative of each of the following functions

1. $y = 3\ln\sqrt[3]{t^2+2}$
2. $y = \ln\dfrac{x}{2x-1}$
3. $y = \dfrac{e^x}{x^2}$

1.2 L'Hospital's Rule

Evaluate the following limits using L'Hospital's rule

1. $\lim_{x \to 0} \frac{1 - e^x}{2x}$
2. $\lim_{x \to 0} \frac{x - \sin x}{x}$

1.3 Applications of Derivatives

Find the minima and maxima, the points of inflection, and sketch the graph of the curve below.

$$y = xe^{-x}$$

1.4 Newton's Method

Find a positive root of the following equation.

$$4 \sin x - x = 0$$

(Note: Not all classes cover this material.)

2 Integration

2.1 General Power Rule Integrals

Using the general power rule to evaluate the following integrals

1. $\displaystyle\int e^{2x}\sqrt{1+e^{2x}}\,dx$

2. $\displaystyle\int (1-\cos 5x)^3 \sin 5x\,dx$

3. $\displaystyle\int_1^e \frac{\sqrt{\ln x}}{x}\,dx$

2.2 Logarithmic and Exponential Integrals

Evaluate the following integrals

1. $\int te^{t^2}\,dx$
2. $\int 2^x\,dx$
3. $\int \frac{1}{x\ln x}\,dx$

2.3 Trigonometric Integrals

Evaluate the following integrals

1. $\int x^2 \sec x^3 \tan x^3 \, dx$
2. $\int x^3 \sec x^4 \, dx$

Evaluate the following integrals

1. $\int \sin^5 x \cos^6 x \, dx$

2. $\int \sin^2 x \cos^2 x \, dx$

2.4 Inverse Trigonometric Forms

Use trigonometric substitution to evaluate the following integral

$$\int \frac{1}{\sqrt{5-3x^2}}\,dx$$

2.5 Trigonometric Substitution

Use a trigonometric substitution to evaluate the following integral.

$$\int \frac{\sqrt{x^2-9}}{x}\,dx$$

2.6 Integration by Parts

Utilize integration by parts to evaluate the following integrals

1. $\int xe^{-x}\,dx$
2. $\int \cot^{-1} x\,dx$

2.7 Integration of Rational Functions

Use either long division or partial fractions to evaluate the following integral.

$$\int \frac{x^3 + 3x}{(x^2 + 1)^2} \, dx$$

3 Series

3.1 Geometric Series

Find the sum of the following series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^{n-1}}$$

3.2 Tests for Convergence

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

(Note: Not all classes cover this material.)

3.3 Maclaurin Series

Find the first three nonzero terms in the Maclaurin series for the following functions

1. $y = \cos x$
2. $y = \ln(1 + x)$

3.4 Operations with Series

Find the Maclaurin series of the following function.

$$y = \ln(1 + x^2)$$

3.5 Computations with Series

Use the first 4 terms of the Maclaurin series for $y = e^{-x}$ to approximate the value of $e^{-0.2}$. Determine the error of your approximation.

3.6 Fourier Series

Determine the Fourier series for the following function on the given interval

$$f(t) = \begin{cases} 0 \text{ if } -1 < t \leq 0 \\ t \text{ if } 0 < t < 1 \end{cases}$$

4 First-Order Differential Equations

4.1 Solutions to Differential Equations

Show that the function

$$y = xe^{-2x} + 3e^{-2x}$$

is a solution to the given differential equation.

$$\frac{dy}{dx} + 2y = e^{-2x}$$

4.2 Separation of Variables

Find the general solution to the given differential equations

1. $dx + (2\cos^2 x - y\cos^2 x)\,dy = 0$
2. $xe^y\,dx + e^{-x}\,dy = 0$

4.3 First-Order Linear Differential Equations

Find the solution to the following differential equation.

$$2\frac{dy}{dx} - 8xy = e^{2x^2}$$

4.4 Applications of Differential Equations

A bacteria culture is known to increase at a rate proportional to the number of bacteria present. It is observed that the size of the culture triples in 3 hours. After how many hours should it be 10 times as large?

5 Higher Order Differential Equations

5.1 Higher-Order Homogeneous Differential Equations

Find the general solution to the given differential equations

1. $6\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$
2. $2D^2y - 3Dy + y = 0$

5.2 Auxiliary Equations

Solve the following differential equations.

1. $(D^2 + 25)y = 0$
2. $(D^2 - 3D + 5)y = 0$

5.3 Non-homogeneous Differential Equations

Find the general solution to the given differential equations.

$$(D^2 - D + 2)y = 4e^{3x}$$

5.4 Applications of Second-Order Equations

A 2 lb weight stretches a spring 6 in. The weight is pushed 7 in above the equilibrium position and released. Find the motion of the weight as a function of time, assuming no damping.

5.5 Computing the Laplace Transformation

Verify the identity.

$$L\{\sin at\} = \frac{a}{s^2 + a^2}$$

5.6 Computing the Inverse Laplace Transformation

Compute the inverse Laplace transformation of the function.

$$F(s) = \frac{5s}{s^2 + 6}$$

Compute the inverse Laplace transformation of the function.

$$F(s) = \frac{s}{(s-1)(s+3)}$$

5.7 Solving Differential Equations Using Laplace Transformations

Use Laplace transformations to solve the following differential equation

$$y'' - 4y' + 4y = e^{3t},\ y(0) = 0,\ y'(0) = -2$$