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1 Differentiation

1.1 Review from Math 22100

Find the derivative of each of the following functions

1. \( y = 3 \ln \sqrt{t^2 + 2} \)

2. \( y = \ln \frac{x}{2x - 1} \)

3. \( y = \frac{e^x}{x^2} \)

1.2 L’Hospital’s Rule

Evaluate the following limits using L’Hospital’s rule

1. \( \lim_{x \to 0} \frac{1 - e^x}{2x} \)

2. \( \lim_{x \to 0} \frac{x - \sin x}{x} \)

1.3 Applications of Derivatives

Find the minima and maxima, the points of inflection, and sketch the graph of the curve below.

\( y = xe^{-x} \)

1.4 Newton’s Method

Find a positive root of the following equation.

\( 4 \sin x - x = 0 \)

(Note: Not all classes cover this material.)

2 Integration

2.1 General Power Rule Integrals

Using the general power rule to evaluate the following integrals

1. \( \int e^{2x} \sqrt{1 + e^{2x}} \, dx \)

2. \( \int (1 - \cos 5x)^3 \sin 5x \, dx \)

3. \( \int_1^e \frac{\sqrt{\ln x}}{x} \, dx \)
2.2 Logarithmic and Exponential Integrals
Evaluate the following integrals
1. $\int te^{t^2} \, dx$
2. $\int 2^x \, dx$
3. $\int \frac{1}{x \ln x} \, dx$

2.3 Trigonometric Integrals
Evaluate the following integrals
1. $\int x^2 \sec x^3 \tan x^3 \, dx$
2. $\int x^3 \sec x^4 \, dx$

Evaluate the following integrals
1. $\int \sin^5 x \cos^6 x \, dx$
2. $\int \sin^2 x \cos^2 x \, dx$

2.4 Inverse Trigonometric Forms
Use trigonometric substitution to evaluate the following integral
$$\int \frac{1}{\sqrt{5 - 3x^2}} \, dx$$

2.5 Trigonometric Substitution
Use a trigonometric substitution to evaluate the following integral.
$$\int \frac{\sqrt{x^2 - 9}}{x} \, dx$$

2.6 Integration by Parts
Utilize integration by parts to evaluate the following integrals
1. $\int xe^{-x} \, dx$
2. $\int \cot^{-1} x \, dx$

2.7 Integration of Rational Functions
Use either long division or partial fractions to evaluate the following integral.
$$\int \frac{x^3 + 3x}{(x^2 + 1)^2} \, dx$$
3 Series

3.1 Geometric Series

Find the sum of the following series.

\[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^{n-1}} \]

3.2 Tests for Convergence

Determine whether the following series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \]

(Note: Not all classes cover this material.)

3.3 Maclaurin Series

Find the first three nonzero terms in the Maclaurin series for the following functions

1. \( y = \cos x \)
2. \( y = \ln(1 + x) \)

3.4 Operations with Series

Find the Maclaurin series of the following function.

\( y = \ln(1 + x^2) \)

3.5 Computations with Series

Use the first 4 terms of the Maclaurin series for \( y = e^{-x} \) to approximate the value of \( e^{-0.2} \). Determine the error of your approximation.

3.6 Fourier Series

Determine the Fourier series for the following function on the given interval

\[ f(t) = \begin{cases} 0 & \text{if } -1 < t \leq 0 \\ t & \text{if } 0 < t < 1 \end{cases} \]

4 First-Order Differential Equations

4.1 Solutions to Differential Equations

Show that the function

\( y = xe^{-2x} + 3e^{-2x} \)

is a solution to the given differential equation.

\[ \frac{dy}{dx} + 2y = e^{-2x} \]
4.2 Separation of Variables
Find the general solution to the given differential equations
1. \( dx + (2 \cos^2 x - y \cos^2 x) \, dy = 0 \)
2. \( xe^y \, dx + e^{-x} \, dy = 0 \)

4.3 First-Order Linear Differential Equations
Find the solution to the following differential equation.

\[
2 \frac{dy}{dx} - 8xy = e^{2x^2}
\]

4.4 Applications of Differential Equations
A bacteria culture is known to increase at a rate proportional to the number of bacteria present. It is observed that the size of the culture triples in 3 hours. After how many hours should it be 10 times as large?

5 Higher Order Differential Equations

5.1 Higher-Order Homogeneous Differential Equations
Find the general solution to the given differential equations
1. \( 6 \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 0 \)
2. \( 2D^2 y - 3Dy + y = 0 \)

5.2 Auxiliary Equations
Solve the following differential equations.
1. \( (D^2 + 25)y = 0 \)
2. \( (D^2 - 3D + 5)y = 0 \)

5.3 Non-homogeneous Differential Equations
Find the general solution to the given differential equations.

\[
(D^2 - D + 2)y = 4e^{3x}
\]

5.4 Applications of Second-Order Equations
A 2 lb weight stretches a spring 6 in. The weight is pushed 7 in above the equilibrium position and released. Find the motion of the weight as a function of time, assuming no damping.

5.5 Computing the Laplace Transformation
Verify the identity.

\[
L\{\sin at\} = \frac{a}{s^2 + a^2}
\]
5.6 Computing the Inverse Laplace Transformation

Compute the inverse Laplace transformation of the function.

\[ F(s) = \frac{5s}{s^2 + 6} \]

Compute the inverse Laplace transformation of the function.

\[ F(s) = \frac{s}{(s - 1)(s + 3)} \]

5.7 Solving Differential Equations Using Laplace Transformations

Use Laplace transformations to solve the following differential equation

\[ y'' - 4y' + 4y = e^{3t}, \; y(0) = 0, \; y'(0) = -2 \]