Contents

1 Limits

1.1 Evaluating Limits

1. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

2. $\lim_{x \to \infty} \frac{x^3 + x^2 + 1}{2x^3 + 1}$

3. $\lim_{x \to 2} \frac{x^2 - 6}{x + 2}$

2 Derivatives

2.1 Four-Step Process

Differentiate the following functions using the Four-Step Process.

1. $f(x) = x^2 + x$
2. $g(x) = \sqrt{x}$

Differentiate the following functions.

1. $y = 5x^7 - 8x^6 + x^2 - 3$
2. $y = (4 - 3x^2)^4$
3. $y = \dfrac{2}{\sqrt{x^3 - x}}$
4. $y = \dfrac{\sqrt{x}}{x - 4}$

2.2 Implicit Differentiation

Find $\frac{dy}{dx}$ by implicit differentiation.

$$5x^2y^3 - y^4 = 2x^3$$

2.3 Higher Derivatives

Find the third derivative of the function below.

$$y = x^6 - 2x^5 - x^4$$

3 Applications of Derivatives

3.1 Extreme Values and Curve Sketching

Find the extreme values of each of the following function and sketch the curve.

$$y = x^4 - 2x^2 - 2$$

Find the minima and maxima, the intervals on which the graph is concave up and concave down, and the inflection points of the following function. Then sketch the curve.

$$y = x^2 + \frac{8}{x}$$

3.2 Optimization

Find the point on the curve $y = \frac{1}{4}x^2$ that is closest to the point $(1, 2)$.

A box with an open top is to be made from a rectangular piece of tin by cutting equal squares from the corners and turning up the sides. The piece of tin measures $1m \times 2m$. Find the size of the squares that yields a maximum capacity for the box.

3.3 Differentials

Find dy if $y = \dfrac{1}{\sqrt{x^2 + 1}}$.

4 Derivatives of Transcendental Functions

4.1 Trigonometric Functions

Find the derivative of each of the following functions.

1. $y = \dfrac{x}{\sin 4x}$
2. $y = \cos^2 x^3$
3. $y = \sqrt{\tan 2x}$
4. $y = \dfrac{\sec 4x}{x^2}$

4.2 Inverse Trigonometric Functions

Find the derivative of each of the following functions.

1. $y = \tan^{-1} 3x$
2. $y = \dfrac{x}{\cos^{-1} x}$

4.3 Exponential and Logarithmic Functions

Find the derivative of each of the following functions.

1. $y = \ln\left(\frac{1}{\sqrt{x+2}}\right)$
2. $y = e^{\cot x}$
3. $y = \sin e^{x}$

4.4 L'Hospital's Rule

Evaluate the following limit.

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x}$$

5 Integrals

5.1 Evaluating Integrals

Find the area bounded by the indicated curves and the x-axis.

$$y = x^3 + 1,\ x = 0,\ x = 1$$

Perform the following integrations.

1. $\int (x^{-2/3} + 3x^{1/2})\, dx$

2. $\int (2x^2 + x)^3 (4x + 1)\, dx$

3. $\int \frac{x}{\sqrt{x^2 - 1}}\, dx$

4. $\int_1^8 \sqrt[3]{x}\, dx$

Find the area of the region bounded by the given curves.

$$y = x^2 - 1, \; y = 3$$

5.2 Improper Integrals

Evaluate the following improper integrals.

1. $\displaystyle\int_1^\infty \frac{2}{x^4}\,dx$
2. $\displaystyle\int_0^\infty \frac{x}{(x^2+4)^2}\,dx$

6 Applications of Integrals

6.1 Volumes of Revolution

A region R is bounded by the given curves. Find the volume of the solid of revolution obtained by rotating the region R about the x-axis.

$$y = \frac{1}{3}x,\ y = 0,\ x = 6$$

Find the volume of the solid obtained by revolving the region bounded by the given curves about the y-axis.

$$y = -x^2 + 3x - 2,\ y = 0,\ x = 0,\ x = 2$$

Obtain the volume of the solid obtained by revolving the region bounded by the given curves about the line $x = 1$.

$$y = x^2,\ y = 0,\ x = 0,\ x = 1$$