## Contents

1 Exponential Functions 3  
1.1 Limit 3  
1.2 Derivative 4  
1.3 Integral 5  

2 Logarithmic Functions 6  
2.1 Limit 6  
2.2 Derivative 7  
2.3 Integral 8  

3 Inverse Trigonometric Functions 9  
3.1 Integral 9  

4 l’Hospital’s Rule 10  
4.1 Indeterminate Limit 10  

5 Techniques of Integration 11  
5.1 Integration by Parts 11  
5.2 Trigonometric Integrals 12  
5.3 Trigonometric Substitution 13  
5.4 Partial Fractions Integral 14  
5.5 U-Substitution 15  

6 Improper Integrals 16  
6.1 Infinite Bounds 16  
6.2 Discontinuous Integrands 17  
6.3 Comparison Test 18  

7 Parametric Curves 19  
7.1 Derivatives 19  
7.2 Areas 20  
7.3 Polar Coordinates 21  
7.4 Areas in Polar Coordinates 22  

8 Applications of Integration 23  
8.1 Arc Length 23  
8.2 Parametric Arc Length 24  
8.3 Polar Coordinates Arc Length 25  
8.4 Surface Area of Revolutions 26  
8.5 Surface Area of Revolution Revisited 27  

9 Infinite Sequences and Series 28  
9.1 Sequences 28  
9.2 Series 29  
9.3 Integral Test 30  
9.4 Limit Comparison Test 31  
9.5 Alternating Series 32  
9.6 Root Test 33
1 Exponential Functions

1.1 Limit

Evaluate the following limit:

\[
\lim_{x \to \infty} \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}
\]
1.2 Derivative

Differentiate the function:

\[ y = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]
1.3 Integral

Evaluate:

\[
\int_{0}^{1} \frac{\sqrt{1 + e^{-x}}}{e^x} \, dx
\]
2 Logarithmic Functions

2.1 Limit

Evaluate the following limit:

\[
\lim_{x \to \infty} [\ln(2 + x) - \ln(1 + x)]
\]
2.2 Derivative

Find $y'$ if:

$$x^y = y^x, \quad x, y > 0$$
2.3 Integral

Evaluate:

$$\int \frac{x^2 + x - 1}{x + 1} \, dx$$
3 Inverse Trigonometric Functions

3.1 Integral

Evaluate:

\[ \int \frac{x}{x^4 + 9} \, dx \]
4  l’Hospital’s Rule

4.1  Indeterminate Limit

Find the limit:

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}, \quad a, b \neq 0$$
5 Techniques of Integration

5.1 Integration by Parts

Evaluate:

\[ \int \sin(\ln(x)) \, dx \]
5.2 Trigonometric Integrals

Evaluate:

\[ \int \frac{d\phi}{\cos \phi + 1} \]
5.3 Trigonometric Substitution

Evaluate:

\[ \int_{0}^{\pi} \frac{\cos t}{\sqrt{1 + \sin^2 t}} \, dt \]
5.4 Partial Fractions Integral

Evaluate:

$$\int \frac{dx}{1 + e^x}$$
5.5 U-Substitution

Evaluate:

\[ I = \int \frac{dx}{1 + \sqrt[3]{x}} \]
6 Improper Integrals

6.1 Infinite Bounds

Determine whether the integral converges or diverges. If it converges, evaluate the integral:

\[
\int_0^\infty x^3 e^{-x^4} \, dx
\]
6.2 Discontinuous Integrands

Determine whether the integral converges or diverges. If it converges, evaluate the integral:

\[
\int_{1}^{3} \frac{x}{(2x^2 - 8)^{\frac{3}{2}}} \, dx
\]
6.3  Comparison Test

Determine whether this integral converges or diverges:

\[
\int_{1}^{\infty} \frac{x}{x^3 + 1} \, dx
\]
7 Parametric Curves

7.1 Derivatives

Find the equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.

\[ x = 1 + \sqrt{t}, \quad y = e^{t^2} ; \quad (2,e) \]
7.2 Areas

Find the area enclosed by an ellipse using the following parametric equations:

\[ x = a \cos \theta, \quad y = b \sin \theta, \quad 0 \leq \theta \leq 2\pi \]
7.3 Polar Coordinates

Find the points (in polar coordinates) on the given curve where the tangent line is horizontal or vertical:

\[ r = e^\theta \]
7.4 Areas in Polar Coordinates

Find the area of one loop of the curve:

\[ r = \sin(2\theta) \]
8 Applications of Integration

8.1 Arc Length

Find the arc length of the curve on the given interval:

\[ y = \frac{1}{3}(x^2 + 2)^{3/2}, \quad 0 \leq x \leq 2 \]
8.2 Parametric Arc Length

Find the total length of the astroid with the following parametric equations:

\[ x = a \cos^3 \theta, \quad y = a \sin^3 \theta \]
8.3 Polar Coordinates Arc Length

Find the arc length of the following polar curve:

\[ r = 2(1 + \cos \theta) \]
8.4 Surface Area of Revolutions

Find the expression for the surface area of a sphere with radius $r$. 
8.5  Surface Area of Revolution Revisited

Using parametric equations, find the expression for the surface area of a sphere with radius $r$. 
9 Infinite Sequences and Series

9.1 Sequences

Determine whether the following sequences converge or diverge.

1. \( a_n = \frac{\ln(n)}{\ln(2n)} \)

2. \( b_n = \frac{\sin(2n)}{1 + \sqrt{n}} \)
9.2 Series

Determine whether the following series is convergent or divergent. If it is convergent, find its sum.

\[ \sum_{n=1}^{\infty} \frac{3}{n(n + 3)} \]
9.3 Integral Test

Determine whether the series is convergent or divergent.

\[ \sum_{n=2}^{\infty} \frac{1}{n \ln^2 n} \]
9.4  Limit Comparison Test

Determine whether the following series converges or diverges.

\[
\sum_{n=1}^{\infty} \frac{1}{n(1+1/n)}
\]
9.5 Alternating Series

Determine whether the following series converges or diverges.

\[ \sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n}) \]
9.6  Root Test

Determine whether the following series converges or diverges.

\[ \sum_{n=1}^{\infty} \left( \frac{-2n}{n+1} \right)^{5n} \]
9.7  Ratio Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$$
9.8  Power Series

Find the radius of convergence and the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n - 1)}$$
9.9 More Power Series

Find the radius of convergence and the interval of convergence of the series:

\[ \sum_{n=1}^{\infty} \frac{n}{b^n} (x - a)^n, \quad b > 0 \]
9.10 Maclaurin Series

Find the Maclaurin series for the following function:

\[ f(x) = \sin^2 x \]