Contents

1 Exponential Functions

1.1 Limit

Evaluate the following limit:

$$\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

1.2 Derivative

Differentiate the function:

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

1.3 Integral

Evaluate:

$$\int_0^1 \frac{\sqrt{1+e^{-x}}}{e^x}\,dx$$

2 Logarithmic Functions

2.1 Limit

Evaluate the following limit:

$$\lim_{x\to\infty}[\ln(2+x)-\ln(1+x)]$$

2.2 Derivative

Find y' if:

$$x^y = y^x, \quad x, y > 0$$

2.3 Integral

Evaluate:

$$\int \frac{x^2 + x - 1}{x + 1}\, dx$$

3 Inverse Trigonometric Functions

3.1 Integral

Evaluate:

$$\int \frac{x}{x^4 + 9}\, dx$$

4 l'Hospital's Rule

4.1 Indeterminate Limit

Find the limit:

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}, \quad a,b\neq 0$$

5 Techniques of Integration

5.1 Integration by Parts

Evaluate:

$$\int \sin(\ln(x))\,dx$$

5.2 Trigonometric Integrals

Evaluate:

$$\int \frac{d\phi}{\cos\phi + 1}$$

5.3 Trigonometric Substitution

Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{\cos t}{\sqrt{1+\sin^2 t}}\, dt$$

5.4 Partial Fractions Integral

Evaluate:

$$\int \frac{dx}{1+e^x}$$

5.5 U-Substitution

Evaluate:

$$I = \int \frac{dx}{1 + \sqrt[3]{x}}$$

6 Improper Integrals

6.1 Infinite Bounds

Determine whether the integral converges or diverges. If it converges, evaluate the integral:

$$\int_0^\infty x^3 e^{-x^4}\, dx$$

6.2 Discontinuous Integrands

Determine whether the integral converges or diverges. If it converges, evaluate the integral:

$$\int_1^3 \frac{x}{(2x^2-8)^{\frac{2}{3}}}\,dx$$

6.3 Comparison Test

Determine whether this integral converges or diverges:

$$\int_1^\infty \frac{x}{x^3+1}\,dx$$

7 Parametric Curves

7.1 Derivatives

Find the equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.

$$x = 1 + \sqrt{t}, \quad y = e^{t^2} \ ; \ (2, e)$$

7.2 Areas

Find the area enclosed by an ellipse using the following parametric equations:

$$x = a\cos\theta, \quad y = b\sin\theta, \quad 0 \leq \theta \leq 2\pi$$

7.3 Polar Coordinates

Find the points (in polar coordinates) on the given curve where the tangent line is horizontal or vertical:

$$r = e^{\theta}$$

7.4 Areas in Polar Coordinates

Find the area of one loop of the curve:

$$r = \sin(2\theta)$$

8 Applications of Integration

8.1 Arc Length

Find the arc length of the curve on the given interval:

$$y = \frac{1}{3}(x^2 + 2)^{3/2}, \quad 0 \leq x \leq 2$$

8.2 Parametric Arc Length

Find the total length of the astroid with the following parametric equations:

$$x = a\cos^3\theta, \quad y = a\sin^3\theta$$

8.3 Polar Coordinates Arc Length

Find the arc length of the following polar curve:

$$r = 2(1 + \cos\theta)$$

8.4 Surface Area of Revolutions

Find the expression for the surface area of a sphere with radius r.

8.5 Surface Area of Revolution Revisited

Using parametric equations, find the expression for the surface area of a sphere with radius r.

9 Infinite Sequences and Series

9.1 Sequences

Determine whether the following sequences converge or diverge.

1. $a_n = \dfrac{\ln(n)}{\ln(2n)}$
2. $b_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$

9.2 Series

Determine whether the following series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

9.3 Integral Test

Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$$

9.4 Limit Comparison Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^{(1+1/n)}}$$

9.5 Alternating Series

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty}(-1)^n(\sqrt{n+1}-\sqrt{n})$$

9.6 Root Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$$

9.7 Ratio Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$$

9.8 Power Series

Find the radius of convergence and the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}$$

9.9 More Power Series

Find the radius of convergence and the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n, \quad b>0$$

9.10 Maclaurin Series

Find the Maclaurin series for the following function:

$$f(x) = \sin^2 x$$