

Contents

| | | |
|----------|--|-----------|
| 1 | Exponential Functions | 3 |
| 1.1 | Limit | 3 |
| 1.2 | Derivative | 4 |
| 1.3 | Integral | 5 |
| 2 | Logarithmic Functions | 6 |
| 2.1 | Limit | 6 |
| 2.2 | Derivative | 7 |
| 2.3 | Integral | 8 |
| 3 | Inverse Trigonometric Functions | 9 |
| 3.1 | Integral | 9 |
| 4 | l'Hospital's Rule | 10 |
| 4.1 | Indeterminate Limit | 10 |
| 5 | Techniques of Integration | 11 |
| 5.1 | Integration by Parts | 11 |
| 5.2 | Trigonometric Integrals | 12 |
| 5.3 | Trigonometric Substitution | 13 |
| 5.4 | Partial Fractions Integral | 14 |
| 5.5 | U-Substitution | 15 |
| 6 | Improper Integrals | 16 |
| 6.1 | Infinite Bounds | 16 |
| 6.2 | Discontinuous Integrands | 17 |
| 6.3 | Comparison Test | 18 |
| 7 | Parametric Curves | 19 |
| 7.1 | Derivatives | 19 |
| 7.2 | Areas | 20 |
| 7.3 | Polar Coordinates | 21 |
| 7.4 | Areas in Polar Coordinates | 22 |
| 8 | Applications of Integration | 23 |
| 8.1 | Arc Length | 23 |
| 8.2 | Parametric Arc Length | 24 |
| 8.3 | Polar Coordinates Arc Length | 25 |
| 8.4 | Surface Area of Revolutions | 26 |
| 8.5 | Surface Area of Revolution Revisited | 27 |
| 9 | Infinite Sequences and Series | 28 |
| 9.1 | Sequences | 28 |
| 9.2 | Series | 29 |
| 9.3 | Integral Test | 30 |
| 9.4 | Limit Comparison Test | 31 |
| 9.5 | Alternating Series | 32 |
| 9.6 | Root Test | 33 |

| | | |
|------|-----------------------------|----|
| 9.7 | Ratio Test | 34 |
| 9.8 | Power Series | 35 |
| 9.9 | More Power Series | 36 |
| 9.10 | Maclaurin Series | 37 |

1 Exponential Functions

1.1 Limit

Evaluate the following limit:

$$\lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

1.2 Derivative

Differentiate the function:

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

1.3 Integral

Evaluate:

$$\int_0^1 \frac{\sqrt{1+e^{-x}}}{e^x} dx$$

2 Logarithmic Functions

2.1 Limit

Evaluate the following limit:

$$\lim_{x \rightarrow \infty} [\ln(2 + x) - \ln(1 + x)]$$

2.2 DerivativeFind y' if:

$$x^y = y^x, \quad x, y > 0$$

2.3 Integral

Evaluate:

$$\int \frac{x^2 + x - 1}{x + 1} dx$$

3 Inverse Trigonometric Functions

3.1 Integral

Evaluate:

$$\int \frac{x}{x^4 + 9} dx$$

4 l'Hospital's Rule

4.1 Indeterminate Limit

Find the limit:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}, \quad a, b \neq 0$$

5 Techniques of Integration

5.1 Integration by Parts

Evaluate:

$$\int \sin(\ln(x)) dx$$

5.2 Trigonometric Integrals

Evaluate:

$$\int \frac{d\phi}{\cos \phi + 1}$$

5.3 Trigonometric Substitution

Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{\cos t}{\sqrt{1 + \sin^2 t}} dt$$

5.4 Partial Fractions Integral

Evaluate:

$$\int \frac{dx}{1 + e^x}$$

5.5 U-Substitution

Evaluate:

$$I = \int \frac{dx}{1 + \sqrt[3]{x}}$$

6 Improper Integrals

6.1 Infinite Bounds

Determine whether the integral converges or diverges. If it converges, evaluate the integral:

$$\int_0^{\infty} x^3 e^{-x^4} dx$$

6.2 Discontinuous Integrands

Determine whether the integral converges or diverges. If it converges, evaluate the integral:

$$\int_1^3 \frac{x}{(2x^2 - 8)^{\frac{2}{3}}} dx$$

6.3 Comparison Test

Determine whether this integral converges or diverges:

$$\int_1^{\infty} \frac{x}{x^3 + 1} dx$$

7 Parametric Curves

7.1 Derivatives

Find the equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.

$$x = 1 + \sqrt{t}, \quad y = e^{t^2}; \quad (2, e)$$

7.2 Areas

Find the area enclosed by an ellipse using the following parametric equations:

$$x = a \cos \theta, \quad y = b \sin \theta, \quad 0 \leq \theta \leq 2\pi$$

7.3 Polar Coordinates

Find the points (in polar coordinates) on the given curve where the tangent line is horizontal or vertical:

$$r = e^{\theta}$$

7.4 Areas in Polar Coordinates

Find the area of one loop of the curve:

$$r = \sin(2\theta)$$

8 Applications of Integration

8.1 Arc Length

Find the arc length of the curve on the given interval:

$$y = \frac{1}{3}(x^2 + 2)^{3/2}, \quad 0 \leq x \leq 2$$

8.2 Parametric Arc Length

Find the total length of the astroid with the following parametric equations:

$$x = a \cos^3 \theta, \quad y = a \sin^3 \theta$$

8.3 Polar Coordinates Arc Length

Find the arc length of the following polar curve:

$$r = 2(1 + \cos \theta)$$

8.4 Surface Area of Revolutions

Find the expression for the surface area of a sphere with radius r .

8.5 Surface Area of Revolution Revisited

Using parametric equations, find the expression for the surface area of a sphere with radius r .

9 Infinite Sequences and Series

9.1 Sequences

Determine whether the following sequences converge or diverge.

1. $a_n = \frac{\ln(n)}{\ln(2n)}$

2. $b_n = \frac{\sin(2n)}{1 + \sqrt{n}}$

9.2 Series

Determine whether the following series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

9.3 Integral Test

Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$$

9.4 Limit Comparison Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^{(1+1/n)}}$$

9.5 Alternating Series

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$$

9.6 Root Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1} \right)^{5n}$$

9.7 Ratio Test

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$$

9.8 Power Series

Find the radius of convergence and the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}$$

9.9 More Power Series

Find the radius of convergence and the interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n}{b^n} (x-a)^n, \quad b > 0$$

9.10 Maclaurin Series

Find the Maclaurin series for the following function:

$$f(x) = \sin^2 x$$