Contents

1 Limits

1.1 Basic Factoring Example

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$$

1.2 One-Sided Limit

$$\lim_{x \to 0} \frac{\sqrt{x^2}}{x}$$

1.3 Squeeze Theorem

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

1.4 Rationalizing

$$\lim_{x \to \infty} -x + \sqrt{x^2 + ax}\ \text{, where } a \text{ is a positive constant}$$

1.5 Limits using Trig. Identities

$$\lim_{u \to 0} \frac{\sin(a+u) - \sin(a)}{u}$$

1.6 Limits involving Infinity, Part I

$$\lim_{n \to \infty} \frac{1 + 2 + 3 + \cdots + n}{n^2}$$

Recall that:

$$1 + 2 + 3 + \cdots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

1.7 Limits involving Infinity, Part II

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In the theory of relativity, the mass of a particle with velocity v is given by the equation above. Find the mass as $v \longrightarrow c^-$.

1.8 Continuity

A function f is defined as follows:

$$f(x) = \begin{cases} \sin(x) & \text{if } x \leq \pi \\ ax + b & \text{if } \pi < x \leq 5 \\ x^2 + b & x > 5 \end{cases}$$

where a, b are constants. Determine a, b such that the function $f(x)$ is continuous everywhere.

1.9 Precise definition of a Limit - Linear Case

$$\text{Prove that } \lim_{x \to c}(ax + b) = ac + b$$

2 Derivatives

2.1 Limit Definition, Part I (i.e. x+h definition)

Using the limit definition, find the derivative of $f(x) = \dfrac{1}{1 + \sqrt{x}}$

2.2 Limit Definition, Part II (i.e. x-a defintion)

Using the limit definition, find the derivative of $f(x) = \dfrac{x}{x - 1}$

2.3 Chain and Product Rule

Calculate the derivative of $f(x) = \sin(\cos^2 x) \cdot \cos(\sin^2 x)$

2.4 Quotient Rule

Calculate the derivative of $f(x) = \dfrac{\sin^2 x}{\sin(x^2)}$

2.5 Ex. involving all types of derivative techniques

Calculate the derivative of $f(x) = \dfrac{x \sin^2(2x)}{(1 + x^2)^2}$

2.6 Derivatives w/ Fractional Exponents

Differentiate $\dfrac{x^{1/5}}{1 + x^{-4/5}}$

3 Integrals

3.1 Riemann sums

Express the integral as a limit of Riemann sums, then evaluate.

$$\int_0^2 (x^2 + x + 1)\,dx$$

3.2 U-substitution, basic

$$\int_{-2}^{-4} (x+4)^{10}\,\mathrm{d}x$$

3.3 Indefinite integral, u-substitution, basic manipulation of substitution

$$\int x^3 \sqrt{x^2+1}\,\mathrm{d}x$$

3.4 Definite integral, u-substitution, basic manipulation of substitution

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,\mathrm{d}x$$

3.5 Integrals w/ Symmetric Limits

$$I = \int_{-1}^{1} \frac{\sin(x)}{1+x^2}\,\mathrm{d}x$$

3.6 Fundamental Theorem of Calculus

Take the derivative of $y(x) = \displaystyle\int_{2x}^{3x+1} \sin(t^4)\,\mathrm{d}t$

4 Applications of Derivatives

4.1 Related Rates Part I

Two people start from the same point. One walks east at 3 mi/hr and the other walks northeast 2 mi/hr. How fast is the distance between the people changing after 15 minutes?

4.2 Related Rates Part II

A Ferris wheel with a radius of 10 m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16 m above ground level?

4.3 Differentials

Use differentials to estimate $\sin(1°)$.

4.4 Linear Approximations

Verify the linear approximation around $x = 0$ for the sine function:

$$\sin(x) \approx x$$

4.5 Optimization (Standard)

Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

4.6 Optimization (Difficult)

Show that of all the isosceles triangles with a given perimeter, the one with the greatest area is equilateral.

4.7 Curve Sketching

Sketch the curve $y = 4x - \tan(x)$, $\frac{-\pi}{2} < x < \frac{\pi}{2}$

5 Applications of Integrals

5.1 Area Between Curves

Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

5.2 Volume of Revolution, Disk Method, Part I

Find the volume of the solid obtained by revolving the area enclosed by $f(x) = \sqrt{4 - x^2}$ and $g(x) = 1$ from $0 \leq x \leq \sqrt{3}$ about the x-axis.

5.3 Volume of Revolution, Disk Method, Part II

Using the same region enclosed by the functions from Problem 29, set up, but do not evaluate, the integral expression for volume if the axis of revolution is moved to $y = -1$.

5.4 Volume of Revolution, Shell Method

Find the volume obtained by revolving the region bounded by the curves $x = (y - 3)^2, x = 4$ about $y = 1$.

5.5 Work, Chain Problem

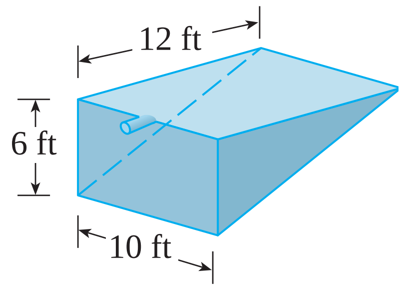
A cable 50 feet in length and weighing 4 pounds per foot (lb/ft) is hanging. Calculate the work done in winding up 25 feet of the cable. Neglect all forces except gravity.

5.6 Work, Spring Problem

A spring has a natural length of 1 meter (m). A force of 100 Newtons compresses it to 0.9 m. How much work is required to compress it to half of its natural length? What is the length of the spring when 20 Joules of work have been expended?

5.7 Work, Involving Geometry

If the tank is filled with water, how much work is required to pump all the water out of the top of the tank? Use the fact that the density of water is 62.5 lb./ft^3.



5.8 Average Value of a Function

Find the average value of $f(x) = |\sin(x)|$ from $0 \leq x \leq 2\pi$.