Contents

1 Real Numbers, Exponents, and Radicals

1.1 Rationalizing the Denominator

Simplify and rationalize the denominator when appropriate.

$$\sqrt[4]{\frac{5x^8y^3}{27x^2}}$$

1.2 Factoring Polynomials

Factor the following polynomials completely.

a. $64x^3 - y^6$

b. $y^2 - x^2 + 8y + 16$

1.3 Algebraic and Fractional Expressions

Simplify the following expression.

$$\frac{(4x^2+9)^{1/2}(2)-(2x+3)(\frac{1}{2})(4x^2+9)^{-1/2}(8x)}{[(4x^2+9)^{1/2}]^2}$$

1.4 Equations

Solve the following equations.

a. $S = \dfrac{p}{q + p(1-q)}$ for q.

b. $\dfrac{2}{2x+1} - \dfrac{3}{2x-1} = \dfrac{-2x+7}{4x^2-1}$.

c. $x = 4 + \sqrt{4x - 19}$

1.5 Apply Problems

In a certain medical test designed to measure carbohydrate tolerance, an adult drinks 7 ounces of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

A farmer plans to close a rectangular region, using part of his barn for one side and fencing for the other three sides. If the side parallel to the barn is to be twice the length of the adjacent side, and the area of the region is to be 128 ft^2, how many feet of fencing should be purchased?

2 Quadratic Equations and Complex Numbers

2.1 Quadratic Equations

a. Solve by completing the square

$$4x^2 - 12x - 11 = 0$$

b. Solve the equation

$$\frac{3}{2}z^2 - 4z - 1 = 0$$

2.2 Complex Numbers

a. Write in the form $a + bi$, where a and b are real numbers.

$$\frac{-4 + 6i}{2 + 7i}$$

b. Find the values of x and y, where x and y are real numbers.

$$(2x - y) - 16i = 10 + 4yi$$

c. Find all solutions to the equation.

$$4x^4 + 25x^2 + 36 = 0$$

2.3 Applied Problems

A baseball is thrown straight upward with an initial speed of $64\frac{ft}{sec}$. The number of feet s above the ground after t seconds is given by the equation

$$s = -16t^2 + 64t$$

a. When will the baseball be 48 feet above the ground?

b. When will it hit the ground?

The recommended distance d that a ladder should be placed away from a vertical wall is 25% of its length L. Approximate the height h that can be reached by relating h as a percentage of L.

2.4 Other Types of Equations

Solve the equation

$$f(x) = 2x^{\frac{-2}{3}} - 7x^{\frac{-1}{3}} - 15 = 0$$

3 Inequalities

3.1 Absolute Values

Solve the equation for x

$$3|x+1|-2=-11$$

3.2 Inequalities and Intervals

Solve and express the solutions in terms of intervals whenever possible.

a. $-\frac{1}{3}|6-5x|+2 \geq -1$

b. $\frac{3}{|5-2x|} < 2$

c. $\frac{x+1}{2x-3} > 2$

4 Functions and Graphs

4.1 Mid-Point

Find an equation for the perpendicular bisector of a line segment AB, where $A = (3, -1)$ and $B = (-2, 6)$.

4.2 Circles

a. Find an equation of the circle where the end points of a diameter are $A(4, -3)$ and $B(-2, 7)$.

b. Find the center and radius of the circle with the given equation

$$2x^2 + 2y^2 - 12x + 4y - 15 = 0$$

4.3 Piecewise Functions

Find the domain and sketch the graph of

$$f(x) = \begin{cases} x + 9 & \text{if } x < -3 \\ -2x & \text{if } |x| \leq 3 \\ -6 & \text{if } x > 3 \end{cases}$$

4.4 Inequality

Solve and express the solution in terms of intervals if possible.

$$\frac{x-2}{x^2-3x-10} \geq 0$$

4.5 Difference Quotient

Simplify the following difference quotient, where $f(x) = x^2 + 5$.

$$\frac{f(x+h) - f(x)}{h} \quad \text{if} \quad h \neq 0$$

4.6 Graphs of Functions

Determine whether f is even, odd, or neither.

$$f(x) = 8x^3 - 3x^2$$

4.7 Parabola

Express $f(x)$ in the form $a(x-h)^2+k$ and sketch a graph of f.

$$f(x) = -3x^2 - 6x - 5$$

4.8 Composite Functions

For $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x^2-16}$ find

a. $(f \circ g)$ and its domain.

b. $(g \circ f)$ and its domain.

4.9 Polynomial Functions of Degree Greater than 2

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and sketch the graph of f.

$$f(x) = x^3 + 2x^2 - 4x - 8$$

5 Properties of Division

5.1 Long Division

Find the quotient and remainder if $f(x) = 3x^3 + 2x - 4$ is divided by $p(x) = 2x^2 + 1$.

5.2 Synthetic Division

Use synthetic division to find the quotient and remainder if $f(x) = 2x^3 - 3x^2 + 4x - 5$ is divided by $p(x) = x - 2$

6 Inverse Functions

6.1 Finding Inverse

Find the inverse function of f.

$$f(x) = \frac{3x+2}{2x-5}$$

6.2 Domain and Range of f^{-1}

Determine the domain and range of f^{-1} for the given function.

$$f(x) = -\frac{4x+5}{3x-8}$$

7 Exponential and Logarithmic Functions

7.1 Exponential Functions

Solve the equation.

(a) $3^{x+4} = 2^{1-3x}$

(b) $2^{2x-3} = 5^{x-2}$

7.2 Compound Interest Formula

If $1000 is invested at a rate of 12% per year compounded monthly, find the amount after

a. 1 month.

b. 6 months.

c. 1 year.

d. 20 years.

7.3 Continuously Compounded Interest Formula

If $P = \$1000$ is deposited in a savings account that pays interest at a rate of 8.25% per year compounded continuously, find the balance after $t = 5$ years.

7.4 Natural Exponential Function

Find the zeros of f.

$$f(x) = x^3(4e^{4x}) + 3x^2e^{4x}$$

The population $N(t)$ (in millions) of the United States t years after 1980 may be approximated by the formula $N(t) = 227e^{0.007t}$. When will the population be twice what it was in 1980?

Use natural logarithms to solve for x in terms of y.

$$y = \frac{e^x - e^{-x}}{2}$$

7.5 Properties of Logarithms

Solve for t using logarithms with base a.

$$A = Ba^{Ct} + D$$

Solve the equation

$$\ln(-4-x)+\ln 3=\ln(2-x)$$

Solve the equation

$$\log_3(x+3) + \log_3(x+5) = 1$$

Find the exact solution, using common logarithms, and a two-decimal-place approximation, when appropriate.

$$\log(x-4) - \log(3x-10) = \log\left(\frac{1}{x}\right)$$

Find the exact solution, using common logarithms, and a two-decimal-place approximation, when appropriate.

$$4^x - 3(4^{-x}) = 8$$

8 Conics

8.1 Parabolas

Find the vertex, focus, and directrix of the parabola. Sketch its graph, showing the focus and the directrix.

$$y = x^2 - 4x + 2$$

Find an equation of the parabola that has its focus at $(6, 4)$ and a directrix of $y = -2$.

Find an equation of the parabola that satisfies the following conditions. A vertex $V(-3, 5)$, axis parallel to the x-axis, and passes through the point $(5, 9)$.

8.2 Ellipses

Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

$$\frac{(x-3)^2}{16} + \frac{(y+4)^2}{9} = 1$$

Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

$$4x^2 + 9y^2 - 32x - 36y + 64 = 0$$

Find an equation of the ellipse that has its center at the origin and satisfies the given conditions.

Vertices $V(\pm 8, 0)$, foci $F(\pm 5, 0)$

8.3 Hyperbolas

Find the vertices, the foci, and the equation of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

$$4y^2 - x^2 + 40y - 4x + 60 = 0$$

Find an equation of the hyperbola that has its center at the origin and satisfies the given conditions.

Vertices $V(\pm 4, 0)$, passing through $(8, 2)$

9 Systems of Equations

9.1 Elimination and Substitution

Use the method of substitution to solve the system.

$$\begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \end{cases}$$

Use the method of substitution to solve the system.

$$\begin{cases} y^2 - 4x^2 = 4 \\ 9y^2 + 16x^2 = 140 \end{cases}$$

Use the method of substitution to solve the system.

$$\begin{cases} x = y^2 - 4y + 5 \\ x - y = 1 \end{cases}$$

9.2 Applied Problems

The price of admission to a high school play was $3.00 for students and $4.50 for nonstudents. If 450 tickets were sold for a total of $1555.50, how many of each kind were purchased?

A small furniture company manufactures sofas and recliners. Each sofa requires 8 hours of labor and $60 in materials, while a recliner can be built for $35 in 6 hours. The company has 340 hours of labor available each week and can afford to buy $2250 worth of materials. How many recliners and sofas can be produced if all labor hours and all materials must be used?

10 Angles and Speeds

10.1 Arcs and Sectors

Given $s = 7$ cm and $r = 4$ cm, answer the following.

a. Find the radian and degree measures of the central angle θ subtended by the given arc of length s on a circle of radius r.

b. Find the area of the sector determined by θ.

10.2 Angular and Linear Speed

Given a radius of 5 inches and 40 rotations per minute (rpm), answer the following.

a. Find the angular speed (in radians per minute).

b. Find the linear speed of a point on the circumference (in ft/min).

11 Values of Trigonometric Functions

11.1 Exact Values

Find the exact values of the following expressions.

a. $\csc\left(\frac{3\pi}{4}\right)$

b. $\csc\left(-\frac{2\pi}{3}\right)$

11.2 Approximate Values

Approximate, to the nearest 0.01 radian, all angles θ in the interval $[0, 2\pi)$ that satisfy the following equations.

a. $\sin(\theta) = 0.4195$

b. $\tan(\theta) = -3.2504$

c. $\sec(\theta) = 1.7452$

11.3 Fundamental Identities

Use fundamental identities to write $\cot(\theta)$ in terms of $\sin(\theta)$, for any acute angle θ.

12 Trigonometric Equations and Identities

12.1 Graphs of Trigonometric Functions

Find the amplitude, period, and phase shift and sketch the graph of the following function.

$$y = -2\sin(3x - \pi)$$

12.2 Applied Problems in Trig.

An airplane takes off at a 10° angle and travels at a rate of 250 ft/sec. Approximately how long does it takes the airplane to reach an altitude of 15,000 feet?

12.3 Verifying Identities

Verify the identity.

$$\sec(\theta) - \cos(\theta) = \tan(\theta)\sin(\theta)$$

Verify the identity.

$$\frac{1}{1-\cos(\gamma)} + \frac{1}{1+\cos(\gamma)} = 2\csc^2(\gamma)$$

Verify the identity.

$$\tan^4(k) - \sec^4(k) = 1 - 2\sec^2(k)$$

12.4 Finding Solutions of Trig. Equations

Find all solutions to the equation.

$$\sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2}$$

Find all solutions that are in the interval $[0, 2\pi)$.

$$2\tan(t) - \sec^2(t) = 0$$

Approximate, to the nearest 10', the solutions in the interval $[0°, 360°)$.

$$\sin^2(t) - 4\sin(t) + 1 = 0$$

13 Inverse Trigonometric Functions And Multiple Angle Formulas

13.1 Double Angle Formulas

Find the exact values of $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ given the information below.

$$\sec(\theta) = -3, \quad 90^\circ < \theta < 180^\circ$$

Use inverse trigonometric functions to find the solutions of the equation that are in $[0, 2\pi)$, approximate to four decimal places.

$$\cos^2(x) + 2\cos(x) - 1 = 0$$

Find the solutions that are in the interval $[0, 2\pi)$.

$$\sin(2t) + \sin(t) = 0$$

13.2 Addition and Subtraction Formulas

If $\sin(\alpha) = -\frac{4}{5}$ and $\sec(\beta) = \frac{5}{3}$ for a third-quadrant angle α and a first-quadrant angle β, find the following.

a. $\sin(\alpha + \beta)$

b. $\tan(\alpha + \beta)$

c. the quadrant containing $\alpha + \beta$

13.3 Inverse Trigonometric Functions

Find the exact value whenever it is defined.

a. $\cot\left(\sin^{-1}\frac{2}{3}\right)$

b. $\sec\left[\tan^{-1}-\left(\frac{3}{5}\right)\right]$

c. $\csc\left[\cos^{-1}-\left(\frac{1}{4}\right)\right]$

Find the exact value whenever it is defined.

a. $\sin\left(\arcsin\frac{1}{2}+\arccos(0)\right)$

b. $\cos\left(\arctan-\frac{3}{4}-\arcsin\frac{4}{5}\right)$

c. $\tan\left(\arctan\frac{4}{3}+\arccos\frac{8}{17}\right)$

Find the exact value whenever it is defined.

a. $\sin\left[2\arccos\left(-\frac{3}{5}\right)\right]$

b. $\cos\left[2\sin^{-1}\left(\frac{15}{17}\right)\right]$

c. $\tan\left[2\tan^{-1}\left(\frac{3}{4}\right)\right]$

Write the following expression as an algebraic expression in x for $x > 0$.

$$\sin(2\sin^{-1} x)$$

14 Law of Sines, Law of Cosines, and Herron's Formula

14.1 Law of Sines

Solve $\triangle ABC$, where $\gamma = 81°$, $c = 11$, and $b = 12$.

A forest ranger at an observation point A sights a fire in the direction $N27°10'E$. Another ranger at an observation point B, 6.0 miles due east of A, sight the same fire at $N52°40'W$. Approximate the distance from A to the fire.

Solve $\triangle ABC$, where $a = 25.0$, $b = 80.0$ and $c = 60.0$.

A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.

14.2 Law of Cosines

Solve $\triangle ABC$, where $\alpha = 80.1°$, $a = 8.0$ and $b = 3.4$.

Approximate the area of the parallelogram that has sides of length $a = 12.0$ and $b = 16.0$ (in feet) if one angle at a vertex has measure $\theta = 40°$.

14.3 Herron's Formula

Approximate the area of $\triangle ABC$, given that $a = 25.0$, $b = 80.0$ and $c = 60.0$.