Contents

1 Conics **2**
- 1.1 Parabolas . . . 2
- 1.2 Ellipses . . . 2
- 1.3 Hyperbolas . . . 2

2 Systems of Equations **2**
- 2.1 Elimination and Substitution . . . 2
- 2.2 Applied Problems . . . 3

3 Angles and Speeds **3**
- 3.1 Arcs and Sectors . . . 3
- 3.2 Angular and Linear Speed . . . 3

4 Values of Trigonometric Functions **3**
- 4.1 Exact Values . . . 3
- 4.2 Approximate Values . . . 4
- 4.3 Fundamental Identities . . . 4

5 Trigonometric Equations and Identities **4**
- 5.1 Graphs of Trigonometric Functions . . . 4
- 5.2 Applied Problems in Trig. . . . 4
- 5.3 Verifying Identities . . . 4
- 5.4 Finding Solutions of Trig. Equations . . . 5

6 Inverse Trigonometric Functions And Multiple Angle Formulas **6**
- 6.1 Double Angle Formulas . . . 6
- 6.2 Addition and Subtraction Formulas . . . 6
- 6.3 Inverse Trigonometric Functions . . . 6

7 Law of Sines, Law of Cosines, and Herron's Formula **7**
- 7.1 Law of Sines . . . 7
- 7.2 Law of Cosines . . . 7
- 7.3 Herron's Formula . . . 7

1 Conics

1.1 Parabolas

(a) Find the vertex, focus, and directrix of the parabola. Sketch its graph, showing the focus and the directrix.

$$y = x^2 - 4x + 2$$

(b) Find an equation of the parabola that satisfies the given conditions.

$$\text{Focus } F(6, 4), \text{ directrix } y = -2$$

(c) Find an equation of the parabola that satisfies the given conditions.

$$\text{Vertex } V(-3, 5), \text{ axis parallel to the } x\text{-axis, and passing through the point (5,9)}$$

1.2 Ellipses

(a) Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

$$\frac{(x-3)^2}{16} + \frac{(y+4)^2}{9} = 1$$

(b) Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

$$4x^2 + 9y^2 - 32x - 36y + 64 = 0$$

(c) Find an equation of the ellipse that has its center at the origin and satisfies the given conditions.

$$\text{Vertices } V(\pm 8, 0), \text{ foci } F(\pm 5, 0)$$

1.3 Hyperbolas

(a) Find the vertices, the foci, and the equation of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

$$4y^2 - x^2 + 40y - 4x + 60 = 0$$

(b) Find an equation of the hyperbola that has its center at the origin and satisfies the given conditions.

$$\text{Vertices } V(\pm 4, 0), \text{ passing through } (8, 2)$$

2 Systems of Equations

2.1 Elimination and Substitution

(a) Use the method of substitution to solve the system.

$$\begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \end{cases}$$

(b) Use the method of substitution to solve the system.

$$\begin{cases} y^2 - 4x^2 = 4 \\ 9y^2 + 16x^2 = 140 \end{cases}$$

(c) Use the method of substitution to solve the system.

$$\begin{cases} x = y^2 - 4y + 5 \\ x - y = 1 \end{cases}$$

2.2 Applied Problems

The price of admission to a high school play was \$3.00 for students and \$4.50 for nonstudents. If 450 tickets were sold for a total of \$1555.50, how many of each kind were purchased?

A small furniture company manufactures sofas and recliners. Each sofa requires 8 hours of labor and \$60 in materials, while a recliner can be built for \$35 in 6 hours. The company has 340 hours of labor available each week and can afford to buy \$2250 worth of materials. How many recliners and sofas can be produced if all labor hours and all materials must be used?

3 Angles and Speeds

3.1 Arcs and Sectors

Given $s = 7cm$ and $r = 4cm$, answer the following.

(a) Find the radian and degree measures of the central angle θ subtended by the given arc of length s on a circle of radius r.

(b) Find the area of the sector determined by θ.

3.2 Angular and Linear Speed

Given a radius of 5 in and 40 rpm, answer the following.

(a) Find the angular speed (in radians per minute).

(b) Find the linear speed of a point on the circumference (in ft/min).

4 Values of Trigonometric Functions

4.1 Exact Values

Find the exact value.

(a) $\csc\left(\frac{3\pi}{4}\right)$

(b) $\csc\left(-\frac{2\pi}{3}\right)$

4.2 Approximate Values

Approximate, to the nearest 0.01 radian, all angles θ in the interval $[0, 2\pi)$ that satisfy the equation.

(a) $\sin(\theta) = 0.4195$

(b) $\tan(\theta) = -3.2504$

(c) $\sec(\theta) = 1.7452$

4.3 Fundamental Identities

Use the fundamental identities to write the first expression in terms of the second, for any acute angle θ.

$$\cot(\theta),\ \sin(\theta)$$

5 Trigonometric Equations and Identities

5.1 Graphs of Trigonometric Functions

Find the amplitude, period, and phase shift and sketching the graph.

$$y = -2\sin(3x - \pi)$$

5.2 Applied Problems in Trig.

An airplane takes off at a 10° angle and travels at a rate of 250ft/sec. Approximately how long does it takes the airplane to reach an altitude of 15,000 feet?

5.3 Verifying Identities

(a) Verify the identity.

$$\sec(\theta) - \cos(\theta) = \tan(\theta)\sin(\theta)$$

(b) Verify the identity.

$$\frac{1}{1-\cos(\gamma)} + \frac{1}{1+\cos(\gamma)} = 2\csc^2(\gamma)$$

(c) Verify the identity.

$$\tan^4(k) - \sec^4(k) = 1 - 2\sec^2(k)$$

5.4 Finding Solutions of Trig. Equations

Find all solutions to the equation.

$$\sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2}$$

Find the solutions that are in the interval $[0, 2\pi)$.

$$2\tan(t) - \sec^2(t) = 0$$

Approximate, to the nearest 10', the solutions in the interval $[0°, 360°)$.

$$\sin^2(t) - 4\sin(t) + 1 = 0$$

6 Inverse Trigonometric Functions And Multiple Angle Formulas

6.1 Double Angle Formulas

Find the exact values of $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ given the information below.

$$\sec(\theta) = -3 \qquad\qquad 90° < \theta < 180°$$

Use inverse trigonometric functions to find the solutions of the equation that are on $[0, 2\pi)$, and approximate solutions to four decimal places.

$$\cos^2(x) + 2\cos(x) - 1 = 0$$

Find the solutions that are in the interval $[0, 2\pi)$.

$$\sin(2t) + \sin(t) = 0$$

6.2 Addition and Subtraction Formulas

If $\sin(\alpha) = -\frac{4}{5}$ and $\sec(\beta) = \frac{5}{3}$ for a third-quadrant angle α and a first-quadrant β, find the following.

(a) $\sin(\alpha + \beta)$

(b) $\tan(\alpha + \beta)$

(c) the quadrant containing $\alpha + \beta$

6.3 Inverse Trigonometric Functions

Find the exact value whenever it is defined.

(a) $\cot\left(\sin^{-1}\frac{2}{3}\right)$

(b) $\sec\left[\tan^{-1} - \left(\frac{3}{5}\right)\right]$

(c) $\csc\left[\cos^{-1} - \left(\frac{1}{4}\right)\right]$

Find the exact value whenever it is defined.

(a) $\sin\left(\arcsin\frac{1}{2} + \arccos(0)\right)$

(b) $\cos\left(\arctan -\frac{3}{4} - \arcsin\frac{4}{5}\right)$

(c) $\tan\left(\arctan\frac{4}{3} + \arccos\frac{8}{17}\right)$

Find the exact value whenever it is defined.

(a) $\sin\left[2\arccos\left(-\frac{3}{5}\right)\right]$

(b) $\cos\left[2\sin^{-1}\left(\frac{15}{17}\right)\right]$

(c) $\tan\left[2\tan^{-1}\left(\frac{3}{4}\right)\right]$

Write the expression as an algebraic expression in x for $x > 0$.

$$\sin(2\sin^{-1} x)$$

7 Law of Sines, Law of Cosines, and Herron's Formula

7.1 Law of Sines

Solve $\triangle ABC$.

$$\gamma = 81^\circ \qquad c = 11 \qquad b = 12$$

A forest ranger at an observation point A sights a fire in the direction $N27^\circ 10' E$. Another ranger at an observation point B, 6.0 miles due east of A, sight the same fire at $N52^\circ 40' W$. Approximate the distance from A to the fire.

Solve $\triangle ABC$.

$$a = 25.0 \qquad b = 80.0 \qquad c = 60.0$$

A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.

7.2 Law of Cosines

Solve $\triangle ABC$.

$$\alpha = 80.1^\circ \qquad a = 8.0 \qquad b = 3.4$$

Approximate the areas of the parallelogram that has sides of length a and b (in feet) if one angle at a vertex has measure θ.

$$a = 12.0 \qquad b = 16.0 \qquad \theta = 40^\circ$$

7.3 Herron's Formula

Approximate the area of $\triangle ABC$.

$$a = 25.0 \qquad b = 80.0 \qquad c = 60.0$$