Contents

1 Real Numbers, Exponents, and Radicals

1.1 Rationalizing the Denominator

Simplify and rationalize the denominator when appropriate.

$$\sqrt[4]{\frac{5x^8y^3}{27x^2}}$$

1.2 Factoring Polynomials

(a) $64x^3 - y^6$

(b) $y^2 - x^2 + 8y + 16$

1.3 Algebraic and Fractional Expressions

Simplify the following expression

$$\frac{(4x^2+9)^{1/2}(2)-(2x+3)(\frac{1}{2})(4x^2+9)^{-1/2}(8x)}{[(4x^2+9)^{1/2}]^2}$$

1.4 Equations

(a) Solve for the specified variable

$$S = \frac{p}{q + p(1-q)} \qquad \text{for } q$$

(b) Solve the equation

$$\frac{2}{2x+1} - \frac{3}{2x-1} = \frac{-2x+7}{4x^2-1}$$

(c) Solve for x

$$x = 4 + \sqrt{4x - 19}$$

1.5 Apply Problems

(a) In a certain medical test designed to measure carbohydrate tolerance, an adult drinks 7 ounces of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%.

i How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

(b) A farmer plans to close a rectangular region, using part of his barn for one side and fencing for the other three sides. If the side parallel to the barn is to be twice the length of the adjacent side, and the area of the region is to be $128 ft^2$,

(i) how many feet of fencing should be purchased?

2 Quadratic Equations and Complex Numbers

2.1 Quadratic Equations

(a) Solve by completing the square

$$4x^2 - 12x - 11 = 0$$

(b) Solve the equation

$$\frac{3}{2}z^2 - 4z - 1 = 0$$

2.2 Complex Numbers

(a) Write in the form $a + bi$, where a and b are real numbers.

$$\frac{-4 + 6i}{2 + 7i}$$

(b) Find the values of x and y, where x and y are real numbers.

$$(2x - y) - 16i = 10 + 4yi$$

(c) Find the solutions to the equation

$$4x^4 + 25x^2 + 36 = 0$$

2.3 Apply Problems

(a) A baseball is thrown straight upward with an initial speed of $64\frac{ft}{sec}$. The number of feet s above the ground after t seconds is given by the equation

$$s = -16t^2 + 64t$$

(i) When will the baseball be 48 feet above the ground?

(ii) When will it hit the ground?

(b) The recommended distance d that a ladder should be placed away from a vertical wall is 25% of its length L. Approximate the height h that can be reached by relating h as a percentage of L.

2.4 Other Types of Equations

Solve the equation

$$f(x) = 2x^{\frac{-2}{3}} - 7x^{\frac{-1}{3}} - 15 = 0$$

3 Inequalities

3.1 Absolute Values

Solve the equation for x

$$3|x+1|-2=-11$$

3.2 Inequalities and Intervals

Solve and express the solutions in terms of intervals whenever possible.

(a) $-\frac{1}{3}|6-5x|+2 \geq -1$

(b) $\frac{3}{|5-2x|} < 2$

(c) $\frac{x+1}{2x-3} > 2$

4 Functions and Graphs

4.1 Mid-Point

Find a general form of an equation for the perpendicular bisector of a segment AB

$$A(3,-1) \qquad\qquad B(-2,6)$$

4.2 Circles

(a) Find an equation of the circle that satisfies the stated conditions.

End points of a diameter $A(4, -3)$ and $B(-2, 7)$

(b) Find the center and radius of the circle with the given equation

$$2x^2 + 2y^2 - 12x + 4y - 15 = 0$$

4.3 Piecewise Functions

Find the domain and sketch the graph of

$$f(x) = \begin{cases} x + 9 & \text{if } x < -3 \\ -2x & \text{if } |x| \leq 3 \\ -6 & \text{if } x > 3 \end{cases}$$

4.4 Inequality

Solve and express the solution in terms of intervals whenever possible.

$$\frac{x-2}{x^2-3x-10} \geq 0$$

4.5 Difference Quotient

Simplify the difference quotient.

$$\frac{f(x+h)-f(x)}{h} \quad \text{if} \quad h \neq 0$$

where $f(x) = x^2 + 5$

4.6 Graphs of Functions

Determine whether f is even, odd, or neither even nor odd.

$$f(x) = 8x^3 - 3x^2$$

4.7 Parabola

Express $f(x)$ in the form $a(x-h)^2+k$ and graph.

$$f(x) = -3x^2 - 6x - 5$$

4.8 Composite Functions

For $f(x)$ and $g(x)$ below find

(a) $(f \circ g)$ and its domain.

(b) $(g \circ f)$ and its domain.

$$f(x) = \sqrt{3 - x} \qquad\qquad g(x) = \sqrt{x^2 - 16}$$

4.9 Polynomial Functions of Degree Greater than 2

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and sketch the graph of f.

$$f(x) = x^3 + 2x^2 - 4x - 8$$

5 Properties of Division

5.1 Long Division

Find the quotient and remainder if $f(x)$ is divided by $p(x)$.

$$f(x) = 3x^3 + 2x - 4 \qquad p(x) = 2x^2 + 1$$

5.2 Synthetic Division

Use synthetic division to find the quotient and remainder if $f(x)$ is divided by $p(x)$

$$f(x) = 2x^3 - 3x^2 + 4x - 5 \qquad p(x) = x - 2$$

6 Inverse Functions

6.1 Finding Inverse

Find the inverse function of f.

$$f(x) = \frac{3x+2}{2x-5}$$

6.2 Domain and Range of f^{-1}

Determine the domain and range of f^{-1} for the given function.

$$f(x) = -\frac{4x+5}{3x-8}$$

7 Exponential and Logarithmic Functions

7.1 Exponential Functions

Solve the equation.

(a) $3^{x+4} = 2^{1-3x}$

(b) $2^{2x-3} = 5^{x-2}$

7.2 Compound Interest Formula

If \$1000 is invested at a rate of 12% per year compounded monthly, find the amount after:

(a) 1 month

(b) 6 months

(c) 1 year

(d) 20 years

7.3 Continuously Compounded Interest Formula

If P dollars is deposited in a savings account that pays interest at a rate of $r\%$ per year compounded continuously, find the balance after t years.

$$P = 1000 \qquad r = 8\frac{1}{4} \qquad t = 5$$

7.4 Natural Exponetial Function

Find the zeros of f.

$$f(x) = x^3(4e^{4x}) + 3x^2e^{4x}$$

The population $N(t)$ (in millions) of the United States t years after 1980 may be approximated by the formula $N(t) = 227e^{0.007t}$.

(a) When will the population be twice what it was in 1980?

Use natural logarithms to solve for x in terms of y.

$$y = \frac{e^x - e^{-x}}{2}$$

7.5 Properties of Logarithms

Solve for t using logarithms with base a

$$A = Ba^{Ct} + D$$

Solve the equation

$$\ln(-4-x)+\ln 3=\ln(2-x)$$

Solve the equation

$$\log_3(x+3) + \log_3(x+5) = 1$$

Find the exact solution, using common logarithms, and a two-decimal-place approximation, when appropriate.

$$\log(x-4) - \log(3x-10) = \log\left(\frac{1}{x}\right)$$

Find the exact solution, using common logarithms, and a two-decimal-place approximation, when appropriate.

$$4^x - 3(4^{-x}) = 8$$