Math 15300 Exam Jam

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1 Chapter 1: Fundamental Concepts of Algebra

Section 1.2 - Exponents and Radicals

Simplify and rationalize the denominator when appropriate.

\[ \sqrt[4]{\frac{5x^8y^3}{27x^2}} \]

Section 1.3 - Algebraic Expressions

(a) \( 64x^3 - y^6 \)

(b) \( y^2 - x^2 + 8y + 16 \)

Section 1.4 - Fractional Expressions

Simplify the following expression

\[ \frac{(4x^2 + 9)^{1/2}(2) - (2x + 3)(\frac{1}{2})(4x^2 + 9)^{-1/2}(8x)}{[(4x^2 + 9)^{1/2}]^2} \]

2 Chapter 2: Equations and Inequalities

Sections 2.1 - Equations

(a) Solve for the specified variable

\[ S = \frac{p}{q + p(1 - q)} \]

for \( q \)

(b) Solve the equation

\[ \frac{2}{2x + 1} - \frac{3}{2x - 1} = \frac{-2x + 7}{4x^2 - 1} \]

Sections 2.2 - Applied Problems

In a certain medical test designed to measure carbohydrate tolerance, an adult drinks 7 ounces of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%.

How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

Section 2.3 - Quadratic Equations

(a) Solve by completing the square

\[ 4x^2 - 12x - 11 = 0 \]

(b) Solve the equation

\[ \frac{3}{2}z^2 - 4z - 1 = 0 \]
Section 2.3 - Applied Problems

1. A farmer plans to close a rectangular region, using part of his barn for one side and fencing for the other three sides. If the side parallel to the barn is to be twice the length of the adjacent side, and the area of the region is to be 128 \text{ft}^2,

(i) how many feet of fencing should be purchased?

2. A baseball is thrown straight upward with an initial speed of 64 \text{ft/sec}. The number of feet \(s\) above the ground after \(t\) seconds is given by the equation

\[ s = -16t^2 + 64t \]

(i) When will the baseball be 48 feet above the ground?
(ii) When will it hit the ground?

3. The recommended distance \(d\) that a ladder should be placed away from a vertical wall is 25% of its length \(L\). Approximate the height \(h\) that can be reached by relating \(h\) as a percentage of \(L\).

Section 2.4 - Complex Numbers

(a) Write in the form \(a + bi\), where \(a\) and \(b\) are real numbers.

\[ \frac{-4 + 6i}{2 + 7i} \]

(b) Find the values of \(x\) and \(y\), where \(x\) and \(y\) are real numbers.

\[ (2x - y) - 16i = 10 + 4yi \]

(c) Find the solutions to the equation

\[ 4x^4 + 25x^2 + 36 = 0 \]

Section 2.5 - Other Types of Equations

1. Solve the equation

\[ f(x) = 2x^{\frac{-2}{3}} - 7x^{\frac{-1}{3}} - 15 = 0 \]

2. Solve for \(x\)

\[ x = 4 + \sqrt{4x - 19} \]

3. Solve the equation for \(x\)

\[ 3|x + 1| - 2 = -11 \]

Sections 2.6 - Inequalities

Solve and express the solutions in terms of intervals whenever possible.

(a) \[ -\frac{1}{3}|6 - 5x| + 2 \geq -1 \]

(b) \[ \frac{3}{|5 - 2x|} < 2 \]
Section 2.7 - More on Inequalities

1. Solve and express the solution in terms of intervals whenever possible.

\[ \frac{x - 2}{x^2 - 3x - 10} \geq 0 \]

2. Solve and express the solution in terms of intervals whenever possible.

\[ \frac{x + 1}{2x - 3} > 2 \]

3 Chapter 3: Functions and Graphs

Section 3.2 - Graphs of Equations

(a) Find an equation of the circle that satisfies the stated conditions.

End points of a diameter \( A(4, -3) \) and \( B(-2, 7) \)

(b) Find the center and radius of the circle with the given equation

\[ 2x^2 + 2y^2 - 12x + 4y - 15 = 0 \]

Section 3.3 - Lines

Find a general form of an equation for the perpendicular bisector of a segment \( AB \)

\[ A(3, -1) \quad B(-2, 6) \]

Section 3.4 - Definition of Function

Simplify the difference quotient.

\[ \frac{f(x + h) - f(x)}{h} \quad \text{if} \quad h \neq 0 \]

where \( f(x) = x^2 + 5 \)

Section 3.5 - Graphs of Functions

1. Find the domain and sketch the graph of

\[ f(x) = \begin{cases} 
  x + 9 & \text{if } x < -3 \\
  -2x & \text{if } |x| \leq 3 \\
  -6 & \text{if } x > 3 
\end{cases} \]

2. Determine whether \( f \) is even, odd, or neither even nor odd.

\[ f(x) = 8x^3 - 3x^2 \]

Section 3.6 - Quadratic Functions

Express \( f(x) \) in the form \( f(x) = a(x - h)^2 + k \) and graph.

\[ f(x) = -3x^2 - 6x - 5 \]
Section 3.7 - Operations on Functions

For \( f(x) \) and \( g(x) \) below find

(a) \( (f \circ g) \) and its domain.
(b) \( (g \circ f) \) and its domain.

\[
f(x) = \sqrt{3 - x} \quad \quad g(x) = \sqrt{x^2 - 16}
\]

4 Chapter 4: Polynomial and Rational Functions

Section 4.1 - Polynomial Functions of Degree Greater than 2

Find all values of \( x \) such that \( f(x) > 0 \) and all \( x \) such that \( f(x) < 0 \), and sketch the graph of \( f \).

\[
f(x) = x^3 + 2x^2 - 4x - 8
\]

Section 4.2 - Properties of Division

1. Find the quotient and remainder if \( f(x) \) is divided by \( p(x) \).
   \[
f(x) = 3x^3 + 2x - 4 \quad \quad p(x) = 2x^2 + 1
\]

2. Use synthetic division to find the quotient and remainder if \( f(x) \) is divided by \( p(x) \).
   \[
f(x) = 2x^3 - 3x^2 + 4x - 5 \quad \quad p(x) = x - 2
\]

5 Chapter 5: Inverse, Exponential, and Logarithmic Functions

Section 5.1 - Inverse Functions

1. Find the inverse function of \( f \).
   \[
f(x) = \frac{3x + 2}{2x - 5}
\]

2. Determine the domain and range of \( f^{-1} \) for the given function.
   \[
f(x) = -\frac{4x + 5}{3x - 8}
\]

Section 5.2 - Exponential Functions

If $1000 is invested at a rate of 7% per year compounded monthly, find the amount after:

(a) 1 month
(b) 6 months
(c) 1 year
(d) 20 years

Section 5.3 - The Natural Exponential Function

1. If \( P \) dollars is deposited in a savings account that pays interest at a rate of \( r\% \) per year compounded continuously, find the balance after \( t \) years.

\[
P = 1000 \quad \quad r = 8\frac{1}{4} \quad \quad t = 5
\]

2. Find the zeros of \( f \).
   \[
f(x) = x^3(4e^{4x}) + 3x^2e^{4x}
\]
Sections 5.4 - Logarithmic Functions

1. The population \( N(t) \) (in millions) of the United States \( t \) years after 1980 may be approximated by the formula \( N(t) = 231e^{0.0103t} \).

   (a) When will the population be twice what it was in 1980?

2. Solve for \( t \) using logarithms with base \( a \)

   \[ A = Ba^{Ct} + D \]

Sections 5.5 - Properties of Logarithms

1. Solve the equation

   \[ \ln(-4-x) + \ln 3 = \ln(2-x) \]

2. Solve the equation

   \[ \log_{3}(x+3) + \log_{3}(x+5) = 1 \]

Sections 5.6 - Exponential and Logarithmic Equations

1. Use natural logarithms to solve for \( x \) in terms of \( y \).

   \[ y = \frac{e^{x} - e^{-x}}{2} \]

2. Solve the equation.

   (a) \( 3^{x+4} = 2^{1-3x} \)
   (b) \( 2^{2x-3} = 5^{x-2} \)

3. Find the exact solution, using common logarithms, and a two-decimal-place approximation, when appropriate.

   \[ \log(x - 4) - \log(3x - 10) = \log \left( \frac{1}{x} \right) \]

4. Find the exact solution, using common logarithms, and a two-decimal-place approximation, when appropriate.

   \[ 4^{x} - 3(4^{-x}) = 8 \]