Math 11100 Exam Jam Solutions

Contents

1 Linear Inequalities and Absolute Value Equations

Solve the following expressions.

a. $|y+2|-1=10$

$$|y+2|-1=10$$
$$|y+2|=11$$

$$y+2=11 \qquad y+2=-11$$
$$\boxed{y=9} \qquad \boxed{y=-13}$$

b. $|2x-1|\geq 7$

$$2x-1\geq 7 \qquad 2x-1\leq -7$$
$$2x\geq 8 \qquad 2x\leq -6$$
$$x\geq 4 \qquad x\leq -3$$

$$\boxed{\text{The solution set is } (-\infty,-3]\cup[4,\infty).}$$

c. $|x+5|-6\leq\ -1$

$$|x+5|-6\leq\ -1$$
$$|x+5|\leq 5$$

$$x+5\leq 5 \qquad x+5\geq -5$$
$$x\leq 0 \qquad x\geq -10$$

$$\boxed{\text{The solution set is } [-10,0].}$$

Solve and graph the solutions for the following problems. Also write the solutions in interval notation.

a. $4 - 3x \geq 10$ **or** $5x - 2 > 13$

$$\begin{aligned} 4 - 3x &\geq 10 \\ -3x &\geq 6 \\ x &\leq -2 \end{aligned}$$

$$\begin{aligned} 5x - 2 &> 13 \\ 5x &> 15 \\ x &> 3 \end{aligned}$$

$$\boxed{(-\infty, -2] \cup (3, \infty)}$$

b. $7x + 4 \geq -17$ **and** $6x + 5 \geq -7$

$$\begin{aligned} 7x + 4 &\geq -17 \\ 7x &\geq -21 \\ x &\geq -3 \end{aligned}$$

$$\begin{aligned} 6x + 5 &\geq -7 \\ 6x &\geq -12 \\ x &\geq -2 \end{aligned}$$

$$\boxed{[-2, \infty)}$$

2 Linear Equations, Graphing and Solving Systems of Equations

Find the equation of the line through $(-5, -2)$ that is perpendicular to $-5x - 2y = 27$. Write the equation in slope-intercept form.

$$-5x - 2y = 27$$
$$-2y = 5x + 27$$
$$y = -\frac{5}{2}x - \frac{27}{2}$$

We can see that the slope of this equation is $-\frac{5}{2}$ meaning any equation that is perpendicular has an opposite and reciprocal slope, yielding $\frac{2}{5}$. Now, we can use the point-slope equation to find a line through (-5,-2) with a slope of $\frac{2}{5}$.

Point-slope formula: $y - y_1 = m(x - x_1)$

$$y - (-2) = \frac{2}{5}(x - (-5))$$
$$y + 2 = \frac{2}{5}(x + 5)$$
$$y + 2 = \frac{2x}{5} + 2$$

$$\boxed{y = \frac{2x}{5}}$$

Find the equation of the line that goes through $(-3, 7)$ and $(2, -1)$ and write it in standard form.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 7}{2 - (-3)} = \frac{-8}{5}$$

Point-slope formula: $y - y_1 = m(x - x_1)$

$$\begin{aligned} y - (7) &= \frac{-8}{5}(x - (-3)) \\ y - 7 &= \frac{-8}{5}(x + 3) \\ 5(y - 7) &= -8(x + 3) \\ 5y - 35 &= -8x - 24 \\ 5y + 8x &= -24 + 35 \end{aligned}$$

$$\boxed{8x + 5y = 11}$$

(a.) Solve the following system of equations.

(b.) Is the system consistent or inconsistent? Are the equations dependent or independent?

$$\begin{cases} 9x + 5y = -19 \\ 4x - 3y = 2 \end{cases}$$

$$\begin{cases} 3(9x + 5y) = 3(-19) \\ 5(4x - 3y) = 5(2) \end{cases}$$

$$\begin{cases} 27x + 15y = -57 \\ 20x - 15y = 10 \end{cases}$$

$$\begin{aligned} 27x + 15y &= -57 \\ +20x - 15y &= 10 \\ 47x &= -47 \\ x &= -1. \end{aligned}$$

Plugging this into either of our initial equations yields $y = -2$.

The solution is $(-1, -2)$. The equations are consistent and independent.

(a.) Solve the following system of equations.

(b.) Is the system consistent or inconsistent? Are the equations dependent or independent?

$$\begin{cases} 2x - 7y = -20 \\ -6x + 21y = 15 \end{cases}$$

$$\begin{cases} 3(2x - 7y) = 3(-20) \\ -6x + 21y = 15 \end{cases}$$

$$\begin{cases} 6x - 21y = -60 \\ -6x + 21y = 15 \end{cases}$$

$$\begin{array}{c} 6x - 21y = -60 \\ -6x + 21y = 15 \\ 0 = -45 \end{array}$$

$$\boxed{\text{Since } 0 \neq -45, \text{ there is no solution and the equations are inconsistent and independent.}}$$

Solve each problem.

a. Heather paid \$16 for her phone. Her monthly service fee is \$40. Formulate a linear function to model the cost, $C(t)$, for t months of service, and determine the amount of time required for the total cost to reach \$560.

The costs include the cost of the phone and the per month charge. Regardless of the length of time, the cost will include \$16. If t is in months, then in addition to the \$16, we know that there is a fee of \$40 per month. So $C(t) = 16 + 40t$.

To find the amount of time required for the total cost to reach \$560, we have to set $C(t) = 560$.

$$\begin{aligned} 560 &= 16 + 40t \\ 544 &= 40t \\ 13.6 &= t \end{aligned}$$

It would take 13.6 months for the total cost to reach \$560.

b. Rosanna walks 2 mph slower than Simone. In the time it takes Simone to walk 8 mi, Rosanna walks 5 mi. Find the speed of each person.

Let r equal the speed that Simone walks and $r - 2$ equal the speed that Rosanna walks. Since they take the same amount of time, Simone and Rosanna's times can both equal t.

	Distance	Rate	Time
Rosanna	5	$r - 2$	t
Simone	8	r	t

Set up your equations:

$$\frac{D}{R} = T \qquad \frac{5}{r-2} = t \qquad \frac{8}{r} = t$$

Since both equations are equal to t, we can use substitution to solve for r.

$$\begin{aligned} \frac{5}{r-2} &= \frac{8}{r} \\ r(r-2) \cdot \left(\frac{5}{r-2}\right) &= r(r-2) \cdot \left(\frac{8}{r}\right) \\ 5r &= 8(r-2) \\ 5r &= 8r - 16 \\ -3r &= -16 \\ r &= \frac{16}{3} \\ r &\approx 5.33 \\ r - 2 &\approx 3.33 \end{aligned}$$

Simone walks at approximately 5.33 mph; Rosanna walks at approximately 3.33 mph.

c. A well and a spring are filling a swimming pool. Together they can fill the pool in three hours. The well working alone can fill the pool in 8 hours less time than the spring. How long will the spring take working alone to fill the pool?

The sum of the time it takes to fill the pool together over the individual times of the well and the spring is equal to 1.

Let $t-8=$ the time it takes the well to fill the pool
Let $t=$ the time it takes the spring to fill the pool

$$
\begin{aligned}
\frac{3}{t-8}+\frac{3}{t}&=1\\
t(t-8)\cdot\left(\frac{3}{t-8}+\frac{3}{t}\right)&=t(t-8)\cdot 1\\
3t+3(t-8)&=t(t-8)\\
3t+3t-24&=t^2-8t\\
6t-24&=t^2-8t\\
0&=t^2-14t+24\\
0&=(t-2)(t-12)
\end{aligned}
$$

$t-2=0$
~~$t=2$~~

$t-12=0$
$t=12$

We can eliminate $t=2$ because the well takes 8 hours less than the spring and setting $t=2$ would give the well a time of -6 hours.

Therefore, it would take the spring 12 hours working alone to fill the pool.

Find the domain.

a. $f(x) = \sqrt{4 - 9x}$

We know that the inside of a square root must be positive. Since it is in the numerator, it can also be zero.

$$4 - 9x \geq 0$$
$$-9x \geq -4$$
$$x \leq \tfrac{4}{9}$$

Note that the inequality switched when divided by a negative. Thus, our domain is $\boxed{D: \Big(-\infty, \frac{4}{9}\Big]}$

b. $f(x) = \dfrac{x^3 - x^2 + x + 2}{x^2 + 12x + 35}$

Now our concern for this function is the denominator. So we know that the function will be defined, everything in the denominator is **not** zero.

$$x^2 + 12x + 35 = 0$$
$$(x + 7)(x + 5) = 0$$
$$x = -7, -5$$

Then our domain is $\mathbb{R}$ and $x \neq -5, -7$ or, in intervalic notation, $\boxed{(-\infty, -7) \cup (-7, -5) \cup (-5, \infty)}$.

Solve.

a. Two solutions, one with a concentration of 25% alcohol and another with a concentration of 35% alcohol, are mixed together to form 20 gallons of solution. How many gallons of each should be mixed together if the result is to have a concentration of 32% alcohol?

Let x be the volume of the 25% alcohol solution and y be the volume of the 35% alcohol added to the mixture. Their sum must be 20 gallons, giving us our first of two equations. Multiplying the volumes by their desired concentrations will yield the second as seen below.

$$\begin{cases} x + y = 20 \\ 0.25x + 0.35y = 0.32(20) \end{cases}$$

$$\begin{cases} x + y = 20 \\ 25x + 35y = 640 \end{cases}$$

Using the substitution $y = 20 - x$, we see that:

$$25x + 35(20 - x) = 640$$
$$25x + 700 - 35x = 640$$
$$-10x = -60$$
$$x = 6 \text{ and } y = 14.$$

6 gallons of the 25% solution should be mixed with 14 gallons of the 35% solution.

b. Paint Town sold 45 paintbrushes, one type at \$8.50 each and another type at \$9.75 each. In all, \$398.75 was taken in for the brushes. How many of each type were sold?

First, let x represent the number of paintbrushes sold that cost \$8.50 and y be the number of paintbrushes sold that cost \$9.75. Since the total number of paintbrushes sold was 45, we know that $x + y = 45$. Furthermore, knowing how much each type costs, we can say that $8.50x + 9.75y = 398.75$. We get the following system of equations.

$$\begin{cases} x + y = 45 \\ 8.50x + 9.75y = 398.75 \end{cases}$$

By solving the first equation for x, we can see that $x = 45 - y$. To make things easier to calculate, let us also multiply the second equation by 100 to get rid of the decimals giving us $850x + 975y = 39865$. Now, making the substitution for x, we get:

$$850(45 - y) + 975y = 39875$$
$$38250 - 850y + 975y = 39875$$
$$125y = 1625$$
$$y = 13$$

Now plugging that into $x + y = 45$, we can see that $x = 32$.

Therefore, 32 brushes were sold that cost \$8.50 and 13 were sold that cost \$9.75.

c. A cruise boat travels 72 miles downstream in 4 hours and returns to its starting point upstream in 6 hours. Find the speed of the river.

In this problem, we are given the distance and the time and we are asked to find the speed of the river. Since the cruise boat is traveling in a river, its speed is affected by the current. When it is traveling downstream, the current adds to its speed; when it is traveling upstream, the current subtracts from its speed.

	Distance	Rate	Time
Downstream	72	$r+c$	4
Upstream	72	$r-c$	6

Set up your equations:

$$D = T \cdot R \qquad 72 = 4(r+c) \qquad 72 = 6(r-c)$$

Since both equations are equal to 72, we can use substitution to solve for r.

$$\begin{aligned} 4(r+c) &= 6(r-c) \\ 4r + 4c &= 6r - 6c \\ 10c &= 2r \\ 5c &= r \end{aligned}$$

Now we can use r to solve for c.

$$\begin{aligned} 72 &= 4(r+c) \\ 72 &= 4(5c+c) \\ 72 &= 4(6c) \\ 72 &= 24c \\ 3 &= c \end{aligned}$$

The speed of the river is 3 mph.

3 Polynomials and Rational Expressions

a. Simplify $\dfrac{(3x^5y^{-3})^{-4}}{9xy^2}$.

$$\frac{1}{9xy^2(3^4x^{20}y^{-12})}$$
$$\frac{y^{12}}{9\cdot 3^4x\cdot x^{20}y^2}$$
$$\boxed{\frac{y^{10}}{729x^{21}}}$$

b. Multiply $(3x-7y)^2$.

$$(3x-7y)^2$$
$$9x^2-21xy-21xy+49y^2$$
$$\boxed{9x^2-42xy+49y^2}$$

c. Divide $\dfrac{x^2+3x-10}{x+5}$.

$$\frac{x^2+3x-10}{x+5}$$
$$\frac{(x+5)(x-2)}{x+5}$$
$$\frac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}}$$
$$\boxed{x-2}$$

Factor the following expressions.

a. $64x^9y^9 + 24x^2y^6$

$\boxed{8x^2y^6(8x^7y^3 + 3)}$

b. $m^3 + 4m^2 - 6m - 24$

$m^2(m+4) - 6(m+4)$

$\boxed{(m^2 - 6)(m+4)}$

c. $8x^2 - 6x - 9$

$\boxed{(4x+3)(2x-3)}$

d. $16x^2 - 81$

$(4x)^2 - 9^2$

$\boxed{(4x-9)(4x+9)}$

e. $8c^3 + 125$

$(2c)^3 + 5^3$

$\boxed{(2c+5)(4c^2 - 10c + 25)}$

f. $x^2 + 6x + 9 - 4y^2$

$(x^2 + 6x + 9) - 4y^2$

$(x+3)^2 - (2y)^2$

$[(x+3) + 2y][(x+3) - 2y]$

$\boxed{(x + 2y + 3)(x - 2y + 3)}$

Solve the following equations.

a. $2k^2 = 9k - 9$

$$2k^2 - 9k + 9 = 0$$

Use the quadratic equation to solve for k.

$$k = \frac{9 \pm \sqrt{(-9)^2 - 4(2)(9)}}{2(2)}$$
$$k = \frac{9 \pm \sqrt{9}}{4}$$
$$k = \frac{9 \pm 3}{4}$$

$$k = \frac{9+3}{4} \qquad k = \frac{9-3}{4}$$

$$k = \frac{12}{4} \qquad k = \frac{6}{4}$$

$$\boxed{k = 3} \qquad \boxed{k = \frac{3}{2}}$$

b. $\dfrac{3}{k+2} - \dfrac{2}{k^2-4} = \dfrac{1}{k-2}$

$$\frac{3}{k+2} - \frac{2}{k^2-4} = \frac{1}{k-2}$$
$$\frac{3}{k+2} - \frac{2}{(k+2)(k-2)} = \frac{1}{k-2}$$
$$(k+2)(k-2)\left[\frac{3}{k+2} - \frac{2}{(k+2)(k-2)}\right] = \left[\frac{1}{k-2}\right](k+2)(k-2)$$
$$3(k-2) - 2 = k + 2$$
$$3k - 8 = k + 2$$
$$2k = 10$$

$$\boxed{k = 5}$$

Simplify the following expressions.

a. $\dfrac{m^2-49}{m+1} \div \dfrac{7-m}{m}$

$$\frac{m^2-49}{m+1} \div \frac{7-m}{m}$$

$$\frac{(m+7)(m-7)}{m+1} \cdot -\left(\frac{m}{m-7}\right)$$

$$\frac{(m+7)\cancel{(m-7)}}{m+1} \cdot -\left(\frac{m}{\cancel{m-7}}\right)$$

$$\boxed{-\frac{m(m+7)}{m+1}}$$

b. $\dfrac{5x}{x^2+xy-2y^2} - \dfrac{3x}{x^2+5xy-6y^2}$

$$\frac{5x}{x^2+xy-2y^2} - \frac{3x}{x^2+5xy-6y^2}$$

$$\frac{5x}{(x+2y)(x-y)} - \frac{3x}{(x+6y)(x-y)}$$

$$\frac{(x+6y)}{(x+6y)} \cdot \frac{5x}{(x+2y)(x-y)} - \frac{(x+2y)}{(x+2y)} \cdot \frac{3x}{(x+6y)(x-y)}$$

$$\frac{5x(x+6y)-3x(x+2y)}{(x+6y)(x+2y)(x-y)}$$

$$\frac{5x^2+30xy-3x^2-6y}{(x+6y)(x+2y)(x-y)}$$

$$\frac{2x^2+24xy}{(x+6y)(x+2y)(x-y)}$$

$$\boxed{\frac{2x(x+12y)}{(x+6y)(x+2y)(x-y)}}$$

c. $\dfrac{\frac{x^2-16y^2}{xy}}{\frac{1}{y}-\frac{4}{x}}$

$$\frac{\frac{x^2-16y^2}{xy}}{\frac{1}{y}-\frac{4}{x}}$$

$$\frac{\frac{(x+4y)(x-4y)}{xy}}{\left(\frac{x}{x}\right)\frac{1}{y}-\left(\frac{y}{y}\right)\frac{4}{x}}$$

$$\frac{\frac{(x+4y)(x-4y)}{xy}}{\frac{x-4y}{xy}}$$

$$\frac{\frac{(x+4y)\cancel{(x-4y)}}{\cancel{xy}}}{\frac{\cancel{x-4y}}{\cancel{xy}}}$$

$$\boxed{x+4y}$$

d. $\dfrac{(a+b)}{\frac{a}{ab}-\frac{b}{a^2}}$

$$\frac{(a+b)}{\frac{a}{ab}-\frac{b}{a^2}}$$

$$\frac{(a+b)}{\left(\frac{a}{a}\right)\frac{a}{ab}-\left(\frac{b}{b}\right)\frac{b}{a^2}}$$

$$\frac{(a+b)}{\frac{a^2-b^2}{a^2b}}$$

$$\frac{(a+b)}{1}\cdot\frac{a^2b}{(a^2-b^2)}$$

$$\frac{\cancel{(a+b)}}{1}\cdot\frac{a^2b}{\cancel{(a+b)}(a-b)}$$

$$\boxed{\frac{a^2b}{a-b}}$$

4 Radical Expressions and Rational Numbers as Exponents

Simplify the following expressions.

a. $\sqrt[5]{s^3} \cdot \sqrt[4]{s}$, write the answer in radical notation.

First, write the radicals using exponential notation. Convert the solution back to radical notation.

$$s^{\frac{3}{5}} \cdot s^{\frac{1}{4}}$$
$$s^{\frac{12}{20}} \cdot s^{\frac{5}{20}}$$
$$s^{\frac{17}{20}}$$
$$\boxed{\sqrt[20]{s^{17}}}$$

b. $\sqrt{108}$

$$\sqrt{108}$$
$$\sqrt{3 \cdot 36}$$
$$\boxed{6\sqrt{3}}$$

c. $-\sqrt[3]{-125a^6b^9c^{12}}$

$$-\sqrt[3]{-125a^6b^9c^{12}}$$
$$-\sqrt[3]{(-5)^3(a^2)^3(b^3)^3(c^4)^3}$$
$$-(-5)a^2b^3c^4$$
$$\boxed{5a^2b^3c^4}$$

d. $3x\sqrt[3]{xy^2} - 2\sqrt[3]{8x^4y^2}$

$$3x\sqrt[3]{xy^2} - 2\sqrt[3]{8x^4y^2}$$
$$3x\sqrt[3]{xy^2} - 2\sqrt[3]{(2x)^3xy^2}$$
$$3x\sqrt[3]{xy^2} - 2(2x)\sqrt[3]{xy^2}$$
$$3x\sqrt[3]{xy^2} - 4x\sqrt[3]{xy^2}$$
$$\boxed{-x\sqrt[3]{xy^2}}$$

e. $\sqrt{x^2 - 4x + 4}$, assume that all variables represent positive numbers.

$$\sqrt{x^2 - 4x + 4}$$
$$\sqrt{(x-2)(x-2)}$$
$$\sqrt{(x-2)^2}$$
$$\boxed{x - 2}$$

Rational the denominator for the following expressions.

a. $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{6}-\sqrt{5}}$

$$\frac{\sqrt{2}-\sqrt{3}}{\sqrt{6}-\sqrt{5}} \cdot \left(\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right)$$

$$\frac{(\sqrt{2}-\sqrt{3})(\sqrt{6}+\sqrt{5})}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$$

$$\frac{\sqrt{12}+\sqrt{10}-\sqrt{18}-\sqrt{15}}{\sqrt{36}+\sqrt{30}-\sqrt{30}-\sqrt{25}}$$

$$\frac{2\sqrt{3}+\sqrt{10}-3\sqrt{2}-\sqrt{15}}{6-5}$$

$$\boxed{2\sqrt{3}+\sqrt{10}-3\sqrt{2}-\sqrt{15}}$$

b. $\dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}$

$$\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} \cdot \left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)$$

$$\frac{\sqrt{a^2}-\sqrt{ab}}{\sqrt{a^2}-\sqrt{ab}+\sqrt{ab}-\sqrt{b^2}}$$

$$\boxed{\frac{a-\sqrt{ab}}{a-b}}$$

Multiply.

a. $(3\sqrt{7}+2\sqrt{5})(2\sqrt{7}-4\sqrt{5})$

$$\begin{aligned}(3\sqrt{7}+2\sqrt{5})(2\sqrt{7}-4\sqrt{5})\\ 6\sqrt{49}-12\sqrt{35}+4\sqrt{35}-8\sqrt{25}\\ 6(7)-8\sqrt{35}-8(5)\\ 42-8\sqrt{35}-40\\ \boxed{2-8\sqrt{35}}\end{aligned}$$

b. $(\sqrt{3}-\sqrt{2})^2$

$$\begin{aligned}(\sqrt{3}-\sqrt{2})^2\\ \sqrt{9}-\sqrt{6}-\sqrt{6}+\sqrt{4}\\ 3-2\sqrt{6}+2\\ \boxed{5-2\sqrt{6}}\end{aligned}$$

Solve the following equations.

a. $\sqrt[3]{x-8}+3=0$

$$\begin{aligned}
\sqrt[3]{x-8}+3&=0\\
\sqrt[3]{x-8}&=-3\\
(\sqrt[3]{x-8})^3&=(-3)^3\\
x-8&=-27\\
\boxed{x=-19}
\end{aligned}$$

b. $5=\sqrt{7x-3}$

$$\begin{aligned}
5&=\sqrt{7x-3}\\
(5)^2&=(\sqrt{7x-3})^2\\
25&=7x-3\\
28&=7x\\
\boxed{4=x}
\end{aligned}$$

c. $(2w-1)^{2/3}-w^{1/3}=0$

First, write the equation in radical notation.

$$\begin{aligned}
\sqrt[3]{(2w-1)^2}-\sqrt[3]{w}&=0\\
\sqrt[3]{(2w-1)^2}&=\sqrt[3]{w}\\
(\sqrt[3]{(2w-1)^2})^3&=(\sqrt[3]{w})^3\\
(2w-1)^2&=w\\
4w^2-4w+1&=w\\
4w^2-5w+1&=0\\
(4w-1)(w-1)&=0
\end{aligned}$$

$$4w-1=0 \qquad \boxed{w=\frac{1}{4}}$$

$$w-1=0 \qquad \boxed{w=1}$$

5 Quadratic Equations and Functions

Solve the following equations.

a. $(t+5)^2 = 48$

$$\begin{aligned} (t+5)^2 &= 48 \\ \sqrt{(t+5)^2} &= \pm\sqrt{48} \\ t+5 &= \pm\sqrt{3(16)} \\ t+5 &= \pm 4\sqrt{3} \\ \boxed{t} &\boxed{= -5 \pm 4\sqrt{3}} \end{aligned}$$

b. $y^2 - 14y + 49 = 4$

$$\begin{aligned} y^2 - 14y + 49 &= 4 \\ (y-7)^2 &= 4 \\ \sqrt{(y-7)^2} &= \sqrt{4} \\ y - 7 &= \pm 2 \end{aligned}$$

$$y - 7 = -2$$
$$\boxed{y = 5}$$

$$y - 7 = 2$$
$$\boxed{y = 9}$$

Find the value of c such that $9x^2 - 30x + c = 0$ has exactly one solution.

To find the value of c such that $9x^2 - 30x + c = 0$ has exactly one solution, the discriminant must be equal to zero. The discriminant of a quadratic equation is $b^2 - 4ac$.

$$\begin{aligned} (-30)^2 - 4(9)c &= 0 \\ 900 - 36c &= 0 \\ -36c &= -900 \\ \boxed{c = 25} \end{aligned}$$

For the quadratic function, $f(x) = -2x^2 - 2x + 3$, find the following:

a. The vertex is $\boxed{\left(-\frac{1}{2}, \frac{7}{2}\right)}$.

According to the formula $f(x) = ax^2 + bx + c$, the x-coordinate of the vertex can be found using the equation $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{2}{2(-2)}$$
$$= -\frac{1}{2}$$

The y-coordinate of the vertex can be found by plugging in $-\frac{1}{2}$ in for x.

$$y = -2\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 3$$
$$y = -2\left(\frac{1}{4}\right) + 1 + 3$$
$$y = -\frac{1}{2} + 4$$
$$y = \frac{7}{2}$$

b. The line of symmetry is $\boxed{x = -\frac{1}{2}}$.

The line of symmetry is a vertical line at the x-coordinate of the vertex.

c. The maximum or minimum value: maximum is $\boxed{\frac{7}{2}}$.

Since the parabola opens down, it has a maximum value at the y-coordinate of the vertex.

d. The x-intercepts are $\boxed{-\frac{1}{2} \pm \frac{\sqrt{7}}{2}}$.

To find the x-intercepts, plug in 0 for y. We will need to use the quadratic equation to solve for x.

$$0 = -2x^2 - 2x + 3$$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(-2)(3)}}{2(-2)}$$
$$x = \frac{2 \pm \sqrt{28}}{-4}$$
$$x = \frac{2 \pm 2\sqrt{7}}{-4}$$
$$x = \frac{1 \pm \sqrt{7}}{-2}$$
$$x = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}$$

e. The y-intercept is $\boxed{3}$.

To find the y-intercept, plug in 0 for x and solve for y.

$$y = -2(0)^2 - 2(0) + 3$$
$$y = 3$$

f. The graph of the function

A club swimming pool is 30 feet long. The area of the pool is 1200 ft^2. The club members want a paved walkway in a strip of uniform width around the pool. They have enough material to cover 296 ft^2. How wide can the strip be?

Since we know the length and the area of the pool, we can also determine the width.

$$A = l \cdot w$$
$$1200 = 30 \cdot w$$
$$w = 40$$

Now we know that the pool that is 30 ft by 40 ft and the pathway surrounds the pool. We have to subtract the area of the pool from the area contained within the boundary of the walkway to find the total area of the walkway. Let x equal the width of the walkway.

$$\begin{aligned}(30+2x)(40+2x)-1200 &= 296\\ 1200+60x+80x+4x^2-1200 &= 296\\ 4x^2+140x &= 296\\ 4x^2+140x-296 &= 0\\ x^2+35x-74 &= 0\\ (x+37)(x-2) &= 0\end{aligned}$$

$$x+37=0 \qquad\qquad x-2=0$$
$$\cancel{x=-37} \qquad\qquad x=2$$

Since the width of the walkway cannot be negative, the walkway can only be 2 feet wide.

6 The Algebra of Functions, Composite Functions and Inverse Functions

Simplify as much as possible.

a. Find $\frac{f}{g}$ if $f(x) = \frac{x^2 - 16}{x^2 - 10x + 25}$ and $g(x) = \frac{3x - 12}{x^2 - 3x - 10}$.

$$\frac{f}{g}(x) = \frac{\frac{x^2-16}{x^2-10x+25}}{\frac{3x-12}{x^2-3x-10}} = \frac{x^2-16}{x^2-10x+25} \cdot \frac{x^2-3x-10}{3x-12} = \frac{(x-4)(x+4)}{(x-5)(x-5)} \cdot \frac{(x+2)(x-5)}{3(x-4)}$$

$$= \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-5)}(x-5)} \cdot \frac{(x+2)\cancel{(x-5)}}{3\cancel{(x-4)}} = \boxed{\frac{(x+4)(x+2)}{3(x-5)}}$$

b. Find $(f-g)(x)$ if $f(x) = \frac{5ab}{a^2-b^2}$ and $g(x) = \frac{a-b}{a+b}$.

$$(f-g)(x) = f(x) - g(x) = \frac{5ab}{a^2-b^2} - \frac{a-b}{a+b} = \frac{5ab}{(a-b)(a+b)} - \frac{a-b}{a+b} = \frac{5ab}{(a-b)(a+b)} - \frac{(a-b)(a-b)}{(a+b)(a-b)}$$

$$= \frac{5ab-(a-b)^2}{(a-b)(a+b)} = \frac{5ab-(a^2-2ab+b^2)}{(a-b)(a+b)} = \frac{5ab-a^2+2ab-b^2}{(a-b)(a+b)} = \boxed{\frac{-a^2+7ab-b^2}{(a-b)(a+b)}}$$

c. Find $(f \cdot g)(-3)$ if $f(x) = \frac{3x}{6x^2-13x-5}$ and $g(x) = 4x - 10$.

$$f(-3) = \frac{3(-3)}{6(-3)^2-13(-3)-5}$$
$$f(-3) = \frac{-9}{54+39-5}$$
$$f(-3) = \frac{-9}{88}$$

$$g(-3) = 4(-3) - 10$$
$$g(-3) = -12 - 10$$
$$g(-3) = -22$$

$$(f \cdot g)(-3) = \frac{-9}{88} \cdot -22$$
$$(f \cdot g)(-3) = \frac{198}{88}$$
$$\boxed{(f \cdot g)(-3) = 2.25}$$

Determine whether or not $g(x) = \sqrt{x-3}$ is one-to-one and, if possible, find g^{-1}.

Since every value of x leads to exactly one value of y, we can determine that $g(x) = \sqrt{x-3}$ is one-to-one. To find the inverse of the function, we have to set $g(x) = y$, switch the x and y and solve for y.

$$\begin{aligned} g(x) &= \sqrt{x-3} \\ y &= \sqrt{x-3} \\ x &= \sqrt{y-3} \\ (x)^2 &= (\sqrt{y-3})^2 \\ x^2 &= y-3 \\ y &= x^2+3 \\ &\boxed{g^{-1} = x^2+3} \end{aligned}$$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$ given $f(x) = 4x^2 - 1$ and $g(x) = \dfrac{2}{x}$.

$$(f \circ g)(x) = 4\left(\frac{2}{x}\right)^2 - 1$$

$$(f \circ g)(x) = 4\left(\frac{4}{x^2}\right) - 1$$

$$(f \circ g)(x) = \left(\frac{16}{x^2}\right) - 1\left(\frac{x^2}{x^2}\right)$$

$$\boxed{(f \circ g)(x) = \left(\frac{16 - x^2}{x^2}\right)}$$

$$\boxed{(g \circ f)(x) = \frac{2}{4x^2 - 1}}$$

7 Complex Numbers and Fractions

Simplify $\dfrac{4+3i}{5+3i}$. Write your answer in the form $a+bi$.

$$\frac{4+3i}{5+3i} \cdot \frac{(5-3i)}{(5-3i)}$$

$$\frac{20-12i+15i-9i^2}{25-15i+15i-9i^2}$$

$$\frac{20+3i-9(-1)}{25-9(-1)}$$

$$\frac{29+3i}{34}$$

$$\boxed{\frac{29}{34}+\frac{3}{34}i}$$

Multiply $2i(-4-i)^2$.

$$\begin{aligned}
2i(-4-i)^2 \\
2i(16+8i+i^2) \\
2i(16+8i-1) \\
2i(15+8i) \\
30i+16i^2 \\
\boxed{-16+30i}
\end{aligned}$$

Simplify.

a. i^{42}

$$i^{42}$$
$$(i^4)^{10}(i^2)$$
$$(1)^{10}(-1)$$
$$\boxed{-1}$$

b. i^{17}

$$i^{17}$$
$$(i^4)^4(i)$$
$$(1)^4(i)$$
$$\boxed{i}$$

8 Logarithmic and Exponential Functions

Solve the following equations.

a. $16^{2x+1} = 64^{x+3}$

$$\begin{aligned} 16^{2x+1} &= 64^{x+3} \\ 4^{2(2x+1)} &= 4^{3(x+3)} \\ 2(2x+1) &= 3(x+3) \\ 4x+2 &= 3x+9 \\ &\boxed{x=7} \end{aligned}$$

b. $\log_4(2x+4) = 3$

$$\begin{aligned} \log_4(2x+4) &= 3 \\ 4^3 &= 2x+4 \\ 64 &= 2x+4 \\ 60 &= 2x \\ &\boxed{30 = x} \end{aligned}$$

c. $2^{x+3} = 5^x$

You can solve for x using logs or natural logs.

$$\begin{aligned} 2^{x+3} &= 5^x \\ \log 2^{x+3} &= \log 5^x \\ (x+3)\log 2 &= x \log 5 \\ x+3 &= x\left(\frac{\log 5}{\log 2}\right) \\ x - x\left(\frac{\log 5}{\log 2}\right) &= -3 \\ x\left[1 - \left(\frac{\log 5}{\log 2}\right)\right] &= -3 \\ x &= \frac{-3}{\left[1 - \left(\frac{\log 5}{\log 2}\right)\right]} \end{aligned}$$

$$\begin{aligned} 2^{x+3} &= 5^x \\ (x+3)\ln 2 &= x \ln 5 \\ x\ln 2 + 3\ln 2 &= x\ln 5 \\ x\ln 2 - x\ln 5 &= -3\ln 2 \\ x(\ln 2 - \ln 5) &= -3\ln 2 \\ x &= \frac{-3\ln 2}{(\ln 2 - \ln 5)} \\ x &= \frac{-3\ln 2}{(\ln \frac{5}{2})} \end{aligned}$$

$$\boxed{x \approx 2.2694}$$

d. $\log_2(x) + \log_2(x-7) = 3$

$$\begin{aligned}
\log_2(x) + \log_2(x-7) &= 3 \\
\log_2 x(x-7) &= 3 \\
2^3 &= x(x-7) \\
8 &= x^2 - 7x \\
0 &= x^2 - 7x - 8 \\
0 &= (x-8)(x+1)
\end{aligned}$$

$$x - 8 = 0 \qquad\qquad x + 1 = 0$$
$$x = 8 \qquad\qquad \cancel{x = -1}$$

Since we can't have the log of a negative number, the answer is $x = 8$.

Rewrite the following expression as a single logarithm.

$$3\log_p(x) + \frac{1}{2}\log_p(y) - \frac{3}{2}\log_p(z)$$

$$\begin{aligned} 3\log_p(x) + \frac{1}{2}\log_p(y) - \frac{3}{2}\log_p(z) \\ \log_p(x)^3 + \log_p(y)^{\frac{1}{2}} - \log_p(z)^{\frac{3}{2}} \\ \boxed{\log_p\left(\frac{x^3 y^{\frac{1}{2}}}{z^{\frac{3}{2}}}\right)} \end{aligned}$$

Change the base of the following logarithms and estimate them to four decimal places.

a. $\log_\pi(e)$

$$\log_\pi(e)$$
$$\frac{\log e}{\log \pi}$$
$$\boxed{\approx 0.8736}$$

b. $3\log_6 2.75$

$$3\log_6 2.75$$
$$\frac{\log(2.75)^3}{\log 6}$$
$$\boxed{\approx 1.6938}$$

Graph.

a. $y = \left(\frac{2}{3}\right)^x$

Plug in values for x to find y and plot the points. As we increase the values for x, the value of y becomes a smaller and smaller fraction. In other words, the function approaches zero but there is no value of x that can make $y = 0$.

x	y
-2	$\left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$
-1	$\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$
0	$\left(\frac{2}{3}\right)^{0} = 1$
1	$\left(\frac{2}{3}\right)^{1} = \frac{2}{3}$
2	$\left(\frac{2}{3}\right)^{2} = \frac{4}{9}$

b. $y = \log_2 x$

Plug in values for x to find y and plot the points. All of the values for x are positive because we cannot have a log of zero or a log of a negative number. The function will approach the x-axis as the values for y decrease.

x	y
$\frac{1}{4}$	$y = \log_2 \frac{1}{4} = -2$
$\frac{1}{2}$	$y = \log_2 \frac{1}{2} = -1$
1	$y = \log_2 1 = 0$
2	$y = \log_2 2 = 1$
4	$y = \log_2 4 = 2$

Solve the following problems using the interest formulas.

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuous Compound Interest Formula:

$$A = Pe^{rt}$$

a. What will be the amount A in an account with an initial principal of \$4000 if interest is compounded continuously at a rate of 3.5% for 6 years? Also, how long does it take for the account to double?

Fill in the continuous compound interest formula with the initial principal amount, interest rate, and number of years to solve for A.

$$A = 4000e^{(0.035)(6)}$$

$$\boxed{A \approx \$4,934.71}$$

To find how long it takes for the account to double, we can assume that the amount A is twice that of the initial principle, P. Therefore for any principle amount, $\frac{A}{P} = 2$, so $2 = e^{rt}$. Now, we can plug in the interest rate to solve for t.

$$\begin{aligned} 2 &= e^{0.035t} \\ \ln(2) &= \ln(e^{0.035t}) \\ \ln(2) &= 0.035t \\ \frac{\ln(2)}{0.035} &= t \end{aligned}$$

$$\boxed{t \approx 19.8 \text{ years}}$$

b. A college loan of \$29,000 is made at 3% interest compounded annually. After t years, the amount, A, due is given by the function $A(t) = 29,000(1.03)^t$. If no payments are made, how long will it take for the amount due to reach \$35,000?

To find how long it will take for the amount due to reach \$35,000, we have to set $A(t) = 35,000$ and solve for t.

$$\begin{aligned} A(t) &= 29,000(1.03)^t \\ 35,000 &= 29,000(1.03)^t \\ \frac{35}{29} &= (1.03)^t \\ \ln\left(\frac{35}{29}\right) &= \ln(1.03)^t \\ \ln\left(\frac{35}{29}\right) &= t\ln(1.03) \\ \frac{\ln(\frac{35}{29})}{\ln(1.03)} &= t \\ 6.4 &\approx t \end{aligned}$$

$$\boxed{\text{It would take approximately 6.4 years for the amount due to reach \$35,000.}}$$