Math 11000 Exam Jam Solutions

Contents

1 Algebra Review

1. Evaluate $-x^2 - 7x$ when $x = -3$.

Solution

$$
\begin{aligned}
-x^2 - 7x \\
-(-3)^2 - 7(-3) \\
-(9) + 21 \\
12
\end{aligned}
$$

2. Solve for x.

$$8x - (4x + 5) = 19$$

Solution

$$
\begin{aligned}
8x - (4x + 5) &= 19 \\
8x - 4x - 5 &= 19 \\
4x - 5 &= 19 \\
4x &= 24 \\
x &= 6
\end{aligned}
$$

3. Find the slope and the y-intercept of the equation. Also graph the line.

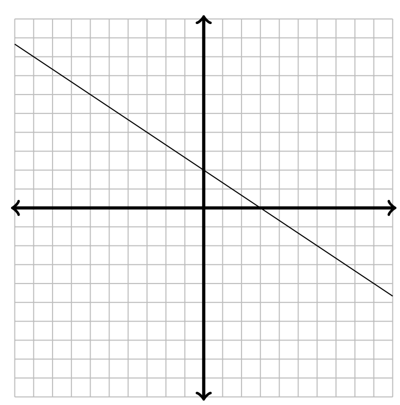
$$2x + 3y = 6$$

Solution

$$
\begin{aligned}
2x + 3y &= 6 \\
3y &= -2x + 6 \\
y &= -\frac{2}{3} + 2
\end{aligned}
$$

slope: $-\frac{2}{3}$

y-intercept: 2



4. Simplify.

$$(8x^4 + 7x^3 - 2) - (2x^3 + x^2 - 3)$$

Solution

$$\begin{aligned}(8x^4 + 7x^3 - 2) - (2x^3 + x^2 - 3)\\ 8x^4 + 7x^3 - 2 - 2x^3 - x^2 + 3\\ 8x^4 + 5x^3 - x^2 + 1\end{aligned}$$

5. Multiply.

$$(2x - 3)^2$$

Solution

$$\begin{aligned}(2x-3)^2\\ (2x-3)(2x-3)\\ 4x^2 - 12x + 9\end{aligned}$$

6. Factor completely. If it is prime, state this.

$$18t^5 - 12t^4 + 6t^3$$

Solution

$$\begin{aligned}18t^5 - 12t^4 + 6t^3\\ 6t^3(3t^2 - 2t + 1)\end{aligned}$$

7. Perform the indicated operation. Then, if possible, simplify.

$$\frac{2-x}{5x^2} \div \frac{x^2-4}{3x}$$

Solution

$$\frac{2-x}{5x^2} \div \frac{x^2-4}{3x}$$

$$\frac{2-x}{5x^2} \cdot \frac{3x}{x^2-4}$$

$$\frac{(-1)(x-2)}{5x^2} \cdot \frac{3x}{(x-2)(x+2)}$$

$$\frac{(-1)\cancel{(x-2)}}{5x\cancel{^2}} \cdot \frac{3\cancel{x}}{\cancel{(x-2)}(x+2)}$$

$$\frac{-3}{5x(x+2)}$$

2 Functions and Graphs

8. Find the function values.

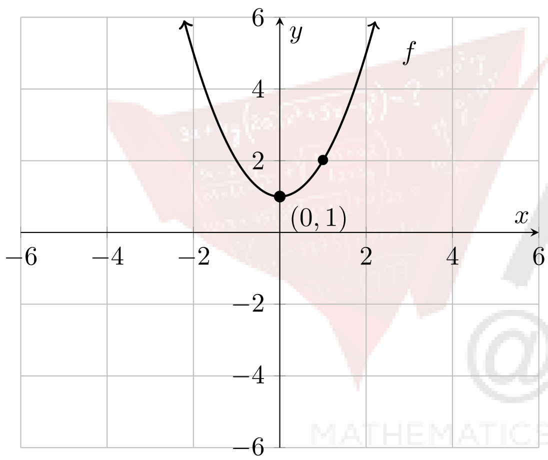
$$f(n) = 5n^2 + 4n$$

(a) $f(-1)$
(b) $f(3)$
(c) $f(2a)$

Solution

(a) $f(-1) = 5(-1)^2 + 4(-1) = 5 - 4 = 1$
(b) $f(3) = 5(3)^2 + 4(3) = 45 + 12 = 57$
(c) $f(2a) = 5(2a)^2 + 4(2a) = 5(4a^2) + 8a = 20a^2 + 8a$

9. For the graph of a function, f, determine the domain and range of f and find $f(1)$ where $f(x) = x^2 + 1$.



Solution

$$\text{Domain: } \mathbb{R}\text{; range: } \{y|y \geq 1\}\text{, or } [1, \infty)$$

$$f(1) = 1^2 + 1 = 1 + 1 = 2$$

10. In 2000, the life expectancy of females born in that year was 79.7 years. In 2010, it was 81.1 years. Let $E(t)$ represent life expectancy and t the number of years since 2000.
    (a) Find a linear function that fits the data.
    (b) Use the linear function of part (a) to predict the life expectancy of females in 2020.

Solution

(a) $E(t) = 0.14t + 79.7$
(b) $E(20) = 0.14(20) + 79.7 = 2.8 + 79.7 = 82.5$ years

11. Let $F(x) = x^2 - 2$ and $G(x) = 5 - x$. Find the following:
    (a) $(F + G)(3)$
    (b) $(F \cdot G)(x)$

Solution

(a) $(F + G)(3)$

$$\begin{aligned}(F+G)(x) &= (x^2-2)+(5-x)\\ &= (3^2-2)+(5-3)\\ &= (9-2)+(2)\\ &= 7+2\\ &= 9\end{aligned}$$

(b) $(F \cdot G)(x)$

$$\begin{aligned}(F\cdot G)(x) &= (x^2-2)(5-x)\\ &= 5x^2-x^3-10+2x\\ &= -x^3+5x^2+2x-10\end{aligned}$$

12. Find the variation constant and an equation of variation if $y = 5$ when $x = 20$ and...
    (a) y varies directly as x.
    (b) y varies inversely as x.

Solution

(a) y varies directly as x.

$$\begin{aligned}y &= kx\\ 5 &= k(20)\\ k &= \frac{1}{4}\end{aligned}$$

(b) y varies inversely as x.

$$\begin{aligned}y &= \frac{k}{x}\\ 5 &= \frac{k}{20}\\ k &= 100\end{aligned}$$

3 Exponents and Radicals

13. Simplify. Variables may represent any real number, so remember to use absolute-value notation when necessary. If a root cannot be simplified, state this.

$$\sqrt{y^2 + 16y + 64}$$

Solution

$$\begin{aligned}\sqrt{y^2 + 16y + 64}\\ \sqrt{(y+8)(y+8)}\\ \sqrt{(y+8)^2}\\ |y+8|\end{aligned}$$

14. Use rational exponents to simplify. Do not use fraction exponents in the final answer. Write the answer using radical notation.

$$\sqrt[12]{a^6}$$

Solution

$$\begin{aligned}\sqrt[12]{a^6}\\ a^{\frac{6}{12}}\\ a^{\frac{1}{2}}\\ \sqrt{a}\end{aligned}$$

15. Simplify. Assume that no radicands were formed by raising negative numbers to even powers.

(a) $\sqrt{45}$

(b) $\sqrt{120}$

(c) $\sqrt{6}\sqrt{33}$

Solution

(a) $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

(b) $\sqrt{120} = \sqrt{4 \cdot 30} = 2\sqrt{30}$

(c) $\sqrt{6}\sqrt{33} = \sqrt{198} = \sqrt{9 \cdot 22} = 3\sqrt{22}$

16. Rationalize the denominator.

$$\sqrt{\frac{5}{8}}$$

Solution

$$\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2\sqrt{4}} = \frac{\sqrt{10}}{2(2)} = \frac{\sqrt{10}}{4}$$

4 Quadratic Functions and Equations

17. Solve by factoring and using the principal of zero products.

(a) $x^2 + 4x - 21 = 0$

(b) $64 + x^2 = 16x$

(c) $4t^2 = 8t$

Solution

(a) $x^2 + 4x - 21 = 0$

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ (x+7)(x-3) &= 0 \\ x + 7 &= 0 \qquad\qquad x - 3 = 0 \\ x &= -7 \qquad\qquad x = 3 \end{aligned}$$

(b) $64 + x^2 = 16x$

$$\begin{aligned} 64 + x^2 &= 16x \\ x^2 - 16x + 64 &= 0 \\ (x-8)(x-8) &= 0 \\ x - 8 &= 0 \\ x &= 8 \end{aligned}$$

(c) $4t^2 = 8t$

$$\begin{aligned} 4t^2 &= 8t \\ 4t^2 - 8t &= 0 \\ 4t(t-2) &= 0 \\ 4t &= 0 \qquad\qquad t - 2 = 0 \\ t &= 0 \qquad\qquad t = 2 \end{aligned}$$

18. Solve for x.

$$4x^2 - 12 = 0$$

Solution

$$\begin{aligned} 4x^2 - 12 &= 0 \\ x^2 - 3 &= 0 \\ x^2 &= 3 \\ \sqrt{x^2} &= \pm\sqrt{3} \\ x &= \pm\sqrt{3} \end{aligned}$$

19. Solve. (Find all complex-number solutions.)

$$(t+5)^2 = 12$$

Solution

$$(t+5)^2 = 12$$
$$\sqrt{(t+5)^2} = \sqrt{12}$$
$$t+5 = \pm\sqrt{12}$$
$$t = -5 \pm \sqrt{4 \cdot 3}$$
$$t = -5 \pm 2\sqrt{3}$$

20. Let $f(x) = 6x^2 - 7x - 20$. Find x such that $f(x) = 0$.

Solution

$$6x^2 - 7x - 20 = 0$$
$$(2x-5)(3x+4) = 0$$

$$2x - 5 = 0$$
$$2x = 5$$
$$x = \frac{5}{2}$$

$$3x + 4 = 0$$
$$3x = -4$$
$$x = -\frac{4}{3}$$

21. A number is 6 less than its square. Find all such numbers.

Solution

$$x = x^2 - 6$$
$$0 = x^2 - x - 6$$
$$0 = (x-3)(x+2)$$

$$x - 3 = 0$$
$$x = 3$$

$$x + 2 = 0$$
$$x = -2$$

22. The distance an object travels in a straight line is given by the function $S(t) = t^2 - 8t$, where S is in feet and t is the number of seconds the object has been in motion. How long does it take the object to move 9 feet?

Solution

$$9 = t^2 - 8t$$
$$t^2 - 8t - 9 = 0$$
$$(t-9)(t+1) = 0$$

$$t - 9 = 0$$
$$t = 9$$

$$t + 1 = 0$$
$$t = -1$$

It would take 9 seconds because $t = -1$ seconds does not exist.

23. Graph the function and find the vertex, the axis of symmetry, and the maximum value or the minimum value.

$$h(x) = -2(x-1)^2 - 3$$

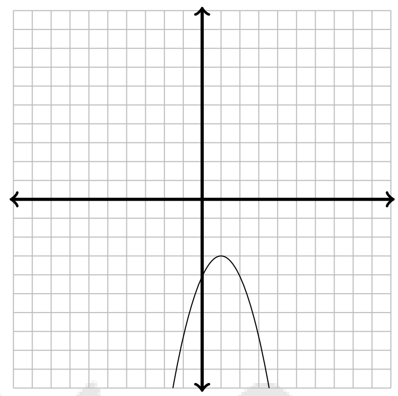
Solution

Vertex: $(1, -3)$; axis of symmetry: $x = 1$; maximum: -3

Plug in values for x to find values for $h(x)$.
Example:

$$h(0) = -2(0-1)^2 - 3 = -2(1) - 3 = -5$$

x	$h(x)$
-1	-11
0	-5
1	-3
2	-11



24. Find any x-intercepts and the y-intercept. If no intercepts exist, state this.

$$f(x) = x^2 - 6x + 3$$

Solution

Use the quadratic formula to solve for the x-intercepts. The y-intercept can be found by plugging in zero for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad f(0) = (0)^2 - 6(0) + 3$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} \qquad f(0) = 3$$

$$x = \frac{6 \pm \sqrt{36 - 12}}{2}$$

$$x = \frac{6pm\sqrt{24}}{2}$$

$$x = \frac{6 \pm 2\sqrt{6}}{2}$$

$$x = 3 \pm \sqrt{6}$$

x-intercepts: $(3 - \sqrt{6}, 0)$, $(3 + \sqrt{6}, 0)$; y-intercept: $(0, 3)$

25. Find the vertex.

$$f(x) = 3x^2 - 12x + 8$$

Solution

$$f(x) = 3x^2 - 12x + 8$$

$$f(x) = ax^2 + bx + c$$
$$V : (h, k)$$
$$h = \frac{-b}{2a}$$
$$k = f(h)$$

$$h = \frac{12}{2(3)} = \frac{12}{6} = 2$$

$$f(2) = 3(2)^2 - 12(2) + 8$$
$$f(2) = 12 - 24 + 8$$
$$f(2) = -4$$
$$V : (2, -4)$$

26. Sweet Harmony Crafts has determined that when x hundred dulcimers are built, the average cost per dulcimer can be estimated by

$$C(x) = 0.1x^2 - 0.7x + 2.425$$

where $C(x)$ is in hundreds of dollars. What is the minimum average cost per dulcimer and how many dulcimers should be built in order to achieve that minimum?

Solution

Solve by finding the coordinates of the vertex.

$$0.1x^2 - 0.7x + 2.425$$

$$h = \frac{-b}{2a} = \frac{0.7}{2(0.1)} = \frac{0.7}{0.2} = 3.5$$
$$C(3.5) = 0.1(3.5)^2 - 0.7(3.5) + 2.425$$
$$0.1(12.25) - 2.45 + 2.425$$
$$1.225 - 0.025$$
$$1.2$$

$$V : (3.5, 1.2)$$

\$120/dulcimer; 350 dulcimers

5 Exponential and Logarithmic Functions

27. Given $f(x) = 5x + 1$ and $g(x) = x^2$, find:
    (a) $(f \circ g)(2)$
    (b) $(g \circ f)(x)$

Solution

(a) $(f \circ g)(2)$

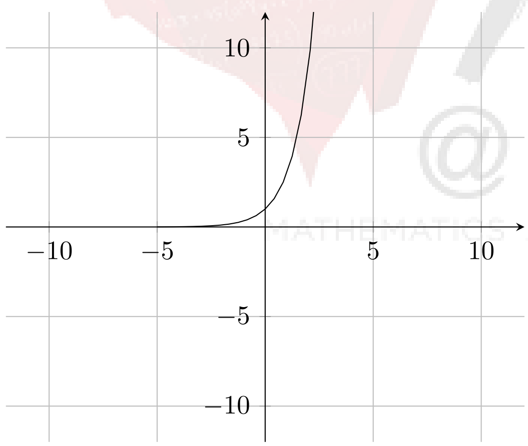
$$g(2) = (2)^2 = 4 \qquad \begin{aligned} f(4) &= 5(4) + 1 \\ f(4) &= 20 + 1 \\ f(4) &= 21 \end{aligned} \qquad (f \circ g)(2) = 21$$

(b) $(g \circ f)(x)$

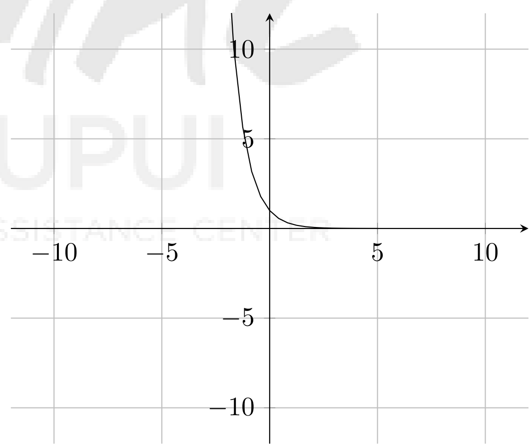
$$\begin{aligned} g(5x + 1) &= (5x + 1)^2 \\ g(5x + 1) &= 25x^2 + 10x + 1 \\ (g \circ f)(x) &= 25x^2 + 10x + 1 \end{aligned}$$

28. Graph.
    (a) $f(x) = 3^x$
    (b) $g(x) = (\frac{1}{4})^x$

Solution



Solution



29. Solve.

$$\log_2 32 = x$$

Solution

$$\begin{aligned} \log_2 32 &= x \\ \frac{\log_{10} 32}{\log_{10} 2} &= x \\ 5 &= x \end{aligned} \qquad \text{OR} \qquad \begin{aligned} \log_2 32 &= x \\ 2^x &= 32 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

30. Express as an equivalent expression, using the individual logarithms of x, y, and z.

$$\log_a \frac{x^5}{y^3 z}$$

Solution

$$\begin{aligned} &\log_a \frac{x^5}{y^3 z} \\ \log_a x^5 - \log_a y^3 &- \log_a z \\ 5\log_a x - 3\log_a y &- \log_a z \end{aligned}$$

31. Use a calculator to find each of the following to four decimal places.

(a) $\log 7$
(b) $\ln 9$
(c) $e^{2.71}$

Solution

(a) 0.8451
(b) 2.1972
(c) 15.0293

32. Solve for x. Approximate to three decimal places if necessary.

(a) $4^{x+1} = 16$
(b) $3^{2x} = 2$
(c) $10^{x-3} = 5$
(d) $6e^{0.05x} = 18$

Solution

(a) $4^{x+1} = 16$

$$\begin{aligned} 4^{x+1} &= 16 \\ 4^{x+1} &= 4^2 \\ x + 1 &= 2 \\ x &= 1 \end{aligned}$$

(b) $3^{2x} = 2$

$$\begin{aligned} 3^{2x} &= 2 \\ \ln(3^{2x}) &= \ln 2 \\ 2x \ln 3 = \ln 2 == 2x &= \frac{\ln 2}{\ln 3} \\ x &= \frac{\ln 2}{2\ln 3} \\ x &\approx 0.3155 \end{aligned}$$

(c) $10^{x-3} = 5$

$$\begin{aligned} 10^{x-3} &= 5 \\ \log(10^{x-3}) &= \log 5 \\ (x-3)(\log 10) &= \log 5 \\ x-3 &= \log 5 \\ x &= 3 + \log 5 \\ x &\approx 3.6990 \end{aligned}$$

(d) $6e^{0.05x} = 18$

$$\begin{aligned} 6e^{0.05x} &= 18 \\ e^{0.05x} &= 3 \\ \ln(e^{0.05x}) &= \ln 3 \\ (0.05x)(\ln e) &= \ln 3 \\ 0.05x &= \ln 3 \\ x &\approx 21.9722 \end{aligned}$$

33. Suppose that P_0 is invested in a savings account where interest is compounded continuously at 3% per year.

(a) Express $P(t)$ in terms of P_0 and 0.03.

(b) Suppose that \$5000 is invested. What is the balance after 1 year? after 2 years?

(c) When will an investment of \$5000 double itself?

Solution

(a) $P(t) = P_0 e^{0.03t}$

(b) 1 year: $P(1) = 5000e^{0.03\cdot 1} = \$5{,}152.27$; 2 years: $P(2) = 5000e^{0.03\cdot 2} = \$5{,}309.18$

(c) $2 = (1)e^{0.03t}$

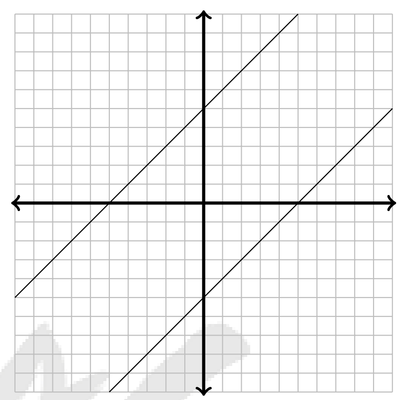
$$\begin{aligned} 2 &= (1)e^{0.03t} \\ \ln 2 &= \ln(e^{0.03t}) \\ \ln 2 &= (0.03t)(\ln e) \\ \ln 2 &= 0.03t \\ \frac{\ln 2}{0.03} &= t \\ 23.1 &\approx t \end{aligned}$$

6 Systems of Linear Equations

34. Solve the system graphically. Be sure to check your solution. If a system has an infinite number of solutions, use set-builder notation to write the solution set. If a system has no solution, state this.

$$\begin{aligned} y - x &= 5, \\ 2x - 2y &= 10 \end{aligned}$$

Solution



No solution

35. Solve using the substitution method.

$$\begin{aligned} 3s - 4t &= 14, \\ 5s + t &= 8 \end{aligned}$$

Solution

$$\begin{aligned} 5s + t &= 8 \\ t &= -5s + 8 \end{aligned}$$

$$\begin{aligned} 3s - 4t &= 14 \\ 3s - 4(-5s + 8) &= 14 \\ 3s + 20s - 32 &= 14 \\ 23s &= 46 \\ s &= 2 \end{aligned}$$

$$\begin{aligned} 5s + t &= 8 \\ 5(2) + t &= 8 \\ 10 + t &= 8 \\ t &= -2 \\ (2&, -2) \end{aligned}$$

36. Ellen wishes to mix candy worth \$1.80 per pound with candy worth \$2.40 per pound to form 48 pounds of a mixture worth \$2.00 per pound. How many pounds of the more expensive candy should she use?

Solution

Amount	Price	Amount $\times$ Price
x	\$1.80	$1.8x$
y	\$2.40	$2.4y$
48	\$2.00	96

$$\begin{aligned} x+y&=48\\ 1.8x+2.4y&=96 \end{aligned}$$

$$\begin{aligned} x&=48-y\\ 1.8(48-y)+2.4y&=96\\ 86.4-1.8y+2.4y&=96\\ 86.4+0.6y&=96\\ 0.6y&=9.6\\ y&=16 \end{aligned}$$

She should use 16 pounds of the more expensive candy.

37. Solve each system. If a system's equations are dependent or if there is no solution, state this.

$$\begin{aligned} x-y-z&=1,\\ 2x+y+2z&=4,\\ x+y+3z&=5 \end{aligned}$$

Solution

$$\begin{array}{rrrcr} x & -y & -z & = & 1\\ 2x & +y & +2z & = & 4\\ \hline 3x & & +z & = & 5 \end{array}$$

$$\begin{array}{rrrcr} x & -y & -z & = & 1\\ x & +y & +3z & = & 5\\ \hline 2x & & +2z & = & 6 \end{array}$$

$$\begin{aligned} -2(3x+z=5)\\ -6x-2z=-10 \end{aligned}$$

$$\begin{array}{rrcr} -6x & -2z & = & -10\\ 2x & +2z & = & 6\\ \hline -4x & & = & -4 \end{array}$$

$$\begin{aligned} -4x&=-4\\ x&=1\\ \\ 3(1)+z&=5\\ z&=2 \end{aligned}$$

$$\begin{aligned} x-y-z&=1\\ 1-y-2&=1\\ -y-1&=1\\ -y&=2\\ y&=-2 \end{aligned}$$

$$(1,-2,2)$$

38. The sum of three numbers is 85. The second is 7 more than the first. The third is 2 more than four times the second. Find the numbers.

Solution

$$\begin{aligned} x_1 + x_2 + x_3 &= 85 \\ x_2 &= x_1 + 7 \\ x_3 &= 4x_2 + 2 \end{aligned}$$

$$\begin{aligned} x_1 + (x_1 + 7) + (4(x_1 + 7) + 2) &= 85 \\ x_1 + x_1 + 7 + 4x_1 + 28 + 2 &= 85 \\ 6x_1 37 &= 85 \\ 6x_1 &= 48 \\ x_1 &= 8 \end{aligned}$$

$$\begin{aligned} x_2 &= 8 + 7 \\ x_2 &= 15 \\ \\ x_3 &= 4(15) + 2 \\ x_3 &= 60 + 2 \\ x_3 &= 62 \end{aligned}$$

8, 15, 62

39. For the following pair of total-cost and total-revenue functions, find the total-profit function and the break-even point.

$$\begin{aligned} C(x) &= 15x + 3100, \\ R(x) &= 40x \end{aligned}$$

Solution

Total-profit function:

$$\begin{aligned} P(x) &= R(x) - C(x) \\ P(x) &= 40x - (15x + 3100) \\ P(x) &= 25x - 3100 \end{aligned}$$

Break-even point:

$$\begin{aligned} R(x) &= C(x) \\ 40x &= 15x + 3100 \\ 25x &= 3100 \\ x &= 124 \end{aligned}$$

$$\begin{aligned} R(124) &= 40(124) \\ R(124) &= 4960 \end{aligned}$$

124 units, \$4960

7 Inequalities

40. Solve algebraically.

$$5(t-3)+4t<2(7+2t)$$

Solution

$$\begin{aligned} 5(t-3)+4t &< 2(7+2t) \\ 5t-15+4t &< 14+4t \\ 9t-15 &< 14+4t \\ 5t &< 29 \\ t &< \frac{29}{5} \end{aligned}$$

$\{t | t < \frac{29}{5}\}$, or $(-\infty, \frac{29}{5})$

41. Find the indicated intersection.

$$\{2,4,16\} \cap \{4,16,256\}$$

Solution

Choose only the elements that the sets have in common.

$$\{4,16\}$$

42. Solve and graph the solution set, where $f(t)=5t+3$.

$$f(t)<-7 \textit{ or } f(t)>8$$

Solution

$$\begin{aligned} f(t) &< -7 \\ 5t+3 &< -7 \\ 5t &< -10 \\ t &< -2 \end{aligned}$$

$$\begin{aligned} f(t) &> 8 \\ 5t+3 &> 8 \\ 5t &> 5 \\ t &> 1 \end{aligned}$$

$\{t | t < -2 \text{ or } t > 1\}$, or $(-\infty,-2) \cup (1,\infty)$

43. Solve and graph $4x - 1 < 7$ and $1 - 3x \leq -5$.

Solution

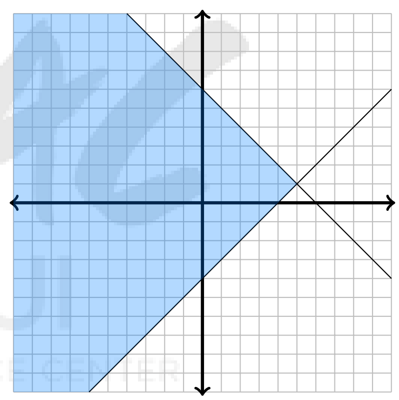
$$\begin{aligned} 4x - 1 &< 7 \\ 4x &< 8 \\ x &< 2 \end{aligned} \qquad \begin{aligned} 1 - 3x &\leq -5 \\ -3x &\leq -6 \\ x &\geq 2 \end{aligned}$$

No solution

44. Graph.

$$\begin{aligned} x + y &\leq 6, \\ x - y &\leq 4 \end{aligned}$$

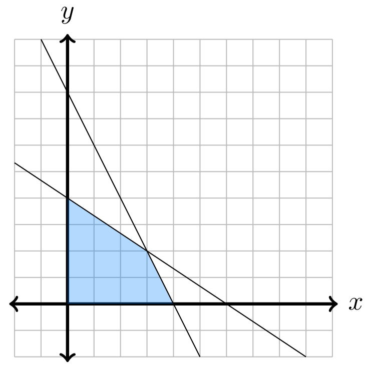
Solution



45. Maximize $F = 6x + 7y$
subject to:

$$\begin{aligned} 2x + 3y &\leq 12 \\ 2x + y &\leq 8 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Solution



Corner Points	Function Values: $F = 6x + 7y$
$(0, 0)$	$F = 6(0) + 7(0) = 0$
$(4, 0)$	$F = 6(4) + 7(0) = 24$
$(3, 2)$	$F = 6(3) + 7(2) = 32$
$(0, 4)$	$F = 6(0) + 7(4) = 28$

Maximum is 32 at $x = 3$ and $y = 2$.

8 Logic and Truth Tables

46. Let p represent the statement "She has green eyes" and let q represent the statement "He is 60 years old." Translate the symbolic compound statement into words.

$$\sim p \vee \sim q$$

Solution

She does not have green eyes or he is not 60 years old.

47. Construct a truth table for the compound statement.

$$(q \vee \sim p) \vee \sim q$$

Solution

p	q	$\sim p$	$q \vee \sim p$	$\sim q$	$(q \vee \sim p) \vee \sim q$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	T	T	T

TTTT

48. Construct a truth table for the statement. Identify whether or not it is a tautology.

$$\sim q \to p$$

Solution

p	q	$\sim q$	$\sim q \to p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

TTTF (Not a tautology)

49. For the given conditional statement, write (a) the converse, (b) the inverse, and (c) the contrapositive in *if ... then* form.

$$p \to \sim q$$

Solution

(a) $\sim q \to p$

(b) $\sim p \to q$

(c) $q \to \sim p$

50. Use a truth table to determine whether the argument is valid or invalid.

$$\begin{array}{c} p \to q \\ \underline{q \to p} \\ p \wedge q \end{array}$$

Solution

p	q	$p \to q$	$q \to p$	$p \wedge q$	$(p \to q) \wedge (q \to p)$	$[(p \to q) \wedge (q \to p)] \to (p \wedge q)$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	F	T	F

invalid

51. Negate the statement: Not all people like football.

Solution

All people like football.

52. Let p represent the statement "Today is Saturday" and let q represent the statement "I will go to the movies." Translate the symbolic compound statement into words.

$$\sim p \vee q, \quad \sim(p \wedge q), \quad p \to q, \text{ and } \sim p \leftrightarrow q$$

Solution

(a) $\sim p \vee q$: Today is not Saturday or I will go to the movies.

(b) $\sim(p \wedge q)$: Today is not Saturday and I will not go to the movies.

(c) $p \to q$: If today is Saturday, then I will go to the movies.

(d) $\sim p \leftrightarrow q$: Today is not Saturday, if and only if I will go to the movies.

53. Use DeMorgan's Laws to negate the statement: It is Saturday and it is not raining.

Solution

Let $p =$ It is Saturday and $q =$ It is raining. Therefore, the statement is $p \wedge \sim q$.
DeMorgan's Laws state:

$$\sim(p \vee q) \equiv \;\sim p \wedge \sim q \quad \text{and} \quad \sim(p \wedge q) \equiv \;\sim p \vee \sim q$$

The negation of $p \wedge \sim q$ is $\sim p \vee q$. The negation in words is "It is not Saturday or it is raining."

54. Write the contrapositive, converse, and inverse of the conditional statement: If I were young, I would be happy.

Solution

(a) Contrapositive: If I were not happy, I would not be young.

(b) Converse: If I were happy, I would be young.

(c) Inverse: If I were not young, I would not be happy.