Math 11000 Exam Jam

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1 Algebra Review

1. Evaluate $-x^2 - 7x$ when $x = -3$.

2. Solve for $x$.

$$8x - (4x + 5) = 19$$

3. Find the slope and the $y$-intercept of the equation. Also graph the line.

$$2x + 3y = 6$$
4. Simplify. 

\[(8x^4 + 7x^3 - 2) - (2x^3 + x^2 - 3)\]

5. Multiply. 

\[(2x - 3)^2\]

6. Factor completely. If it is prime, state this. 

\[18t^5 - 12t^4 + 6t^3\]
7. Perform the indicated operation. Then, if possible, simplify.

\[
\frac{2 - x}{5x^2} \div \frac{x^2 - 4}{3x}
\]

2. Functions and Graphs

8. Find the function values.

\[f(n) = 5n^2 + 4n\]

(a) \(f(-1)\)
(b) \(f(3)\)
(c) \(f(2a)\)

9. For the graph of a function, \(f\), determine the domain and range of \(f\) and find \(f(1)\) where \(f(x) = x^2 + 1\).
10. In 2000, the life expectancy of females born in that year was 79.7 years. In 2010, it was 81.1 years. Let $E(t)$ represent life expectancy and $t$ the number of years since 2000.

(a) Find a linear function that fits the data.
(b) Use the linear function of part (a) to predict the life expectancy of females in 2020.

11. Let $F(x) = x^2 - 2$ and $G(x) = 5 - x$. Find the following:

(a) $(F + G)(3)$
(b) $(F \cdot G)(x)$

12. Find the variation constant and an equation of variation if $y = 5$ when $x = 20$ and...

(a) $y$ varies directly as $x$.
(b) $y$ varies inversely as $x$. 
3 Exponents and Radicals

13. Simplify. Variables may represent any real number, so remember to use absolute-value notation when necessary. If a root cannot be simplified, state this.

\[ \sqrt{y^2 + 16y + 64} \]

14. Use rational exponents to simplify. Do not use fraction exponents in the final answer. Write the answer using radical notation.

\[ \sqrt[12]{a^6} \]

15. Simplify. Assume that no radicands were formed by raising negative numbers to even powers.

(a) \( \sqrt{45} \)

(b) \( \sqrt{120} \)

(c) \( \sqrt{6\sqrt{33}} \)
16. Rationalize the denominator.

\[ \sqrt{\frac{5}{8}} \]

4 Quadratic Functions and Equations

17. Solve by factoring and using the principal of zero products.
   (a) \( x^2 + 4x - 21 = 0 \)
   (b) \( 64 + x^2 = 16x \)
   (c) \( 4t^2 = 8t \)

18. Solve for \( x \).

\[ 4x^2 - 12 = 0 \]
19. Solve. (Find all complex-number solutions.)

\[(t + 5)^2 = 12\]

20. Let \( f(x) = 6x^2 - 7x - 20 \). Find \( x \) such that \( f(x) = 0 \).

21. A number is 6 less than its square. Find all such numbers.
22. The distance an object travels in a straight line is given by the function $S(t) = t^2 - 8t$, where $S$ is in feet and $t$ is the number of seconds the object has been in motion. How long does it take the object to move 9 feet?

23. Graph the function and find the vertex, the axis of symmetry, and the maximum value or the minimum value.

$$h(x) = -2(x - 1)^2 - 3$$

24. Find any $x$-intercepts and the $y$-intercept. If no intercepts exist, state this.

$$f(x) = x^2 - 6x + 3$$
25. Find the vertex.

\[ f(x) = 3x^2 - 12x + 8 \]

26. Sweet Harmony Crafts has determined that when \( x \) hundred dulcimers are built, the average cost per dulcimer can be estimated by

\[ C(x) = 0.1x^2 - 0.7x + 2.425 \]

where \( C(x) \) is in hundreds of dollars. What is the minimum average cost per dulcimer and how many dulcimers should be built in order to achieve that minimum?
5 Exponential and Logarithmic Functions

27. Given $f(x) = 5x + 1$ and $g(x) = x^2$, find:
   (a) $(f \circ g)(2)$
   (b) $(g \circ f)(x)$

28. Graph.
   (a) $f(x) = 3^x$
   (b) $g(x) = \left(\frac{1}{4}\right)^x$

29. Solve.
   \[
   \log_2 32 = x
   \]
30. Express as an equivalent expression, using the individual logarithms of $x$, $y$, and $z$.

$$
\log_a \frac{x^5}{y^3 z}
$$

31. Use a calculator to find each of the following to four decimal places.

(a) $\log 7$
(b) $\ln 9$
(c) $e^{2.71}$
32. Solve for $x$. Approximate to three decimal places if necessary.

(a) $4^{x+1} = 16$
(b) $2^{2x} = 2$
(c) $10^{x-3} = 5$
(d) $6e^{0.05x} = 18$

33. Suppose that $P_0$ is invested in a savings account where interest is compounded continuously at 3% per year.

(a) Express $P(t)$ in terms of $P_0$ and 0.03.
(b) Suppose that $5000 is invested. What is the balance after 1 year? after 2 years?
(c) When will an investment of $5000 double itself?
6 Systems of Linear Equations

34. Solve the system graphically. Be sure to check your solution. If a system has an infinite number of solutions, use set-builder notation to write the solution set. If a system has no solution, state this.

\[ y - x = 5, \]
\[ 2x - 2y = 10 \]

35. Solve using the substitution method.

\[ 3s - 4t = 14, \]
\[ 5s + t = 8 \]
36. Ellen wishes to mix candy worth $1.80 per pound with candy worth $2.40 per pound to form 48 pounds of a mixture worth $2.00 per pound. How many pounds of the more expensive candy should she use?

37. Solve each system. If a system’s equations are dependent or if there is no solution, state this.

\[
\begin{align*}
-x - y - z &= 1, \\
2x + y + 2z &= 4, \\
x + y + 3z &= 5
\end{align*}
\]
38. The sum of three numbers is 85. The second is 7 more than the first. The third is 2 more than four times the second. Find the numbers.

39. For the following pair of total-cost and total-revenue functions, find the total-profit function and the break-even point.

\[ C(x) = 15x + 3100, \]
\[ R(x) = 40x \]
7 Inequalities

40. Solve algebraically.

\[ 5(t - 3) + 4t < 2(7 + 2t) \]

41. Find the indicated intersection.

\[ \{2, 4, 16\} \cap \{4, 16, 256\} \]

42. Solve and graph the solution set, where \( f(t) = 5t + 3 \).

\[ f(t) < -7 \text{ or } f(t) > 8 \]
43. Solve and graph $4x - 1 < 7$ and $1 - 3x \leq -5$.

44. Graph.

\begin{align*}
x + y &\leq 6, \\
x - y &\leq 4
\end{align*}

45. Maximize $F = 6x + 7y$

subject to:

\begin{align*}
2x + 3y &\leq 12 \\
2x + y &\leq 8 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
8 Logic and Truth Tables

46. Let $p$ represent the statement “She has green eyes” and let $q$ represent the statement “He is 60 years old.” Translate the symbolic compound statement into words.

$$\sim p \lor \sim q$$

47. Construct a truth table for the compound statement.

$$(q \lor \sim p) \lor \sim q$$

48. Construct a truth table for the statement. Identify whether or not it is a tautology.

$$\sim q \rightarrow p$$
49. For the given conditional statement, write (a) the converse, (b) the inverse, and (c) the contrapositive in if \ldots then form.

\[ p \rightarrow \sim q \]

50. Use a truth table to determine whether the argument is valid or invalid.

\[
\begin{array}{c}
p \rightarrow q \\
q \rightarrow p \\
p \land q
\end{array}
\]
52. Let $p$ represent the statement “Today is Saturday” and let $q$ represent the statement “I will go to the movies.” Translate the symbolic compound statement into words.

$$\sim p \lor q, \quad \sim (p \land q), \quad p \rightarrow q, \quad \text{and} \quad \sim p \leftrightarrow q$$

53. Use DeMorgan’s Laws to negate the statement: It is Saturday and it is not raining.

54. Write the contrapositive, converse, and inverse of the conditional statement: If I were young, I would be happy.